The Cosmic Computer

The Physics of the Perennial Philosophy

A Yogi's Guide

Gareth Timms

https://www.thecosmiccomputer.org

Quotations

The author thanks the following people for approving the use of their quotations: Oliver Consa, David Hestenes, Tony Hey, Barbara Hubbard, Basil Mahon.

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Although the author sees the quotations used in this book as important to its purpose, their presence should not be taken to indicate any agreement of the quoted people with the ideas presented in this book. Nevertheless, the inclusion of each quotation does show the respect the author feels for the work of each person quoted.

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This book has only been possible thanks to the labours of the many physicists and meditators whose work is cited — long may their search for truth continue.

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About the author

Gareth Timms was educated at Watford Grammar School and St. John's College Cambridge, where he took a B.A. in Engineering. In the early 1980s he and a friend wrote the Desk Top Publishing software *JetSetter*, which was moderately successful until its demise in 1996. Subsequently, he travelled, pursuing his new found passion for yoga and meditation, and this led to the 'Universe as Cosmic Computer' experience that he describes in the Introduction to this book. There followed 21 years of studying physics, seeking understanding...

Review of *The Cosmic Computer*

The true and enduring value of this book is that it gives curious and intelligent readers a severe jolt, makes them sit bolt upright and question all that they have previously understood and taken for granted as perceived wisdom.

The beauty to be found within these pages lies in the imaginative way in which Timms has approached his research. Whilst the reader might not agree with some of his individual claims and propositions, there is a great deal here for the reader to learn from how he has approached the problems that he has tackled. Of all the many threads he is trying to pull together, there are some that the reader might dismiss but there are also many threads that lead to a tangle we all know exists but have yet to understand. This book opens a door that allows the reader to start to untangle those threads.

One can argue that Timms blend of mathematics, science and a diverse range other subjects displays more interdisciplinary dexterity than that of much of today's mainstream scientific community. For that he is to be congratulated.

Having spent my entire career working with students in schools around the world and for 25 years as a Head and schools inspector, this is a book that I would recommend to all students who are engaged in critical thinking outside the box and who seek a better understanding of both themselves and the universe in which they live. As such it is a book I would encourage every school and every university to have several copies of in its library and I, myself, would make it required reading for many applying for university.

Jon Siviter MA (Oxon) (and until retirement C.Phys. F.Inst.P.)

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Introduction

Computation generates reality, it is happening throughout the world around me, inside me, inside every particle, with a power that makes our most advanced computers appear like primitive toys. This was the extraordinary way I experienced the world for ten days in 2000, shortly after passing ten days in silence at a vipassana meditation retreat. I was fortunate that my partner Jacqueline was a psychotherapist, and she kept me sufficiently grounded during my apparent madness to allow the experience to play itself out naturally. A few months later, I watched the recently released film, *The Matrix,* and found the computational reality it depicted uncannily close to my own experience.

Although my experience had its share of delusional madness — my laptop and all my surroundings were communicating with me, and vast amounts of information were downloading into my fevered brain — it also seemed intensely real, far more so than everyday life. It seemed that the veil concealing the underlying computational machinery of the Universe had been lifted and that I was privileged to have been granted this vision. After ten days of this sleepless mental intensity the veil descended again, and with relief, but also sadness, I realised that I was back to nearly normal.

Vipassana meditation was used by Buddha for his own enlightenment and has been practised on his recommendation for over 2500 years by Buddhist monks and yogis, so its effects are well known, and reproducible, although each person's experience is different. The sensation of *more real than reality* is a fairly common experience of meditators, and on a couple of occasions I had already had visions of a pool covered with lotus flowers that had the same super-real quality. Our modern lives are so hectic that on a retreat, it generally takes around three days of continual silence and meditation practice to quieten our distracted monkey minds sufficiently to reach the levels of concentration where effects of this type may appear, the *jhanas*.

I love and trust science, and having escaped at the age of twelve from the brainwashing of my Christian upbringing, I remained very suspicious of religions generally, believing that adoption of a religion required giving up trust in one's own rationality and becoming zombiefied, as many of the faithful seem to me, like children who grow up continuing to believe in Father Christmas. Early retirement from computer programming allowed me the time to travel, following paths of yoga and meditation, and overcoming my distrust of religions by focusing on the essential consensus they all share, the *Perennial Philosophy*. This is brilliantly described by Aldous Huxley in his book *The Perennial Philosophy*, the name adopted from Newton's equally god-believing rival, and co-inventor of the calculus, Wilhelm Leibniz.

Buddhism has been described as a science of the mind rather than a religion. According to Buddha, all events in the world arise from the twelve step mechanical process of dependent origination, *paṭiccasamuppāda*. He seems to be describing a computed reality. Buddha also gave, in his *Kalama Sutta* discourse, a declaration of intellectual independence worthy of any scientist, including the advice of not necessarily believing what is written in a sacred text, or what a teacher tells you — even him.

Unfortunately, like all religions and other human organisations Buddhism has its own distortions and corruptions, and I had been fortunate to learn to meditate at a monastery founded by a Buddhist monk who had become so disillusioned that he went to live in the forest in solitude, and meditated according to Buddha's original instructions. After six years alone, he began to attract followers, founded a forest monastery, and later an international meditation centre available to allcomers. He always encouraged students to find the essential truths in their own religion rather than focus on Buddhism. This is the perspective of the Perennial Philosophy, a distilled consensus of all the great mystics and spiritual masters of the world, stripped of irrational belief in miracles, and infallible texts or persons. We will consider the Perennial Philosophy in Chapter 2.

For much of my life I had felt an uncomfortable split in my psyche, drawn to both science and spirituality, yet aware of their apparent incompatibility. I was repulsed in equal measure by the insistence of physics that the false God of randomness controlled the outcome of quantum processes, as by religious teachings that required me to believe in logical absurdities. Where did I belong? New age friends often seemed overly gullible, ready to believe in anything, while scientific friends seemed blinkered against important aspects of life — there is a wonderful sort of creative magic in the way events in the world play out. The world-as-computation experience gave me the means of healing my split personality by trying to reverse-engineer the world-as-computer that I had 'seen', and gain some understanding of it in terms of known physics.

Study in your own time with a deep desire for understanding is a joy rather than simply work done to get a degree certificate. I followed no fixed course, but was guided by the overall vision, and which topics intuitively 'smelled' important. A principle guide has been the repeated emphasis of the Upanishads and Bhagavad Gita on what I call the dimension of *scale*, locating the One godhead of the Universe at the smallest scale. Another great guide is coincidence. I have often heard physicists say, from their superior viewpoint, that coincidence is just an illusion that more gullible humans are prone to. Having experienced too many powerful examples in my own life, I do not believe them, and there are many extraordinary examples in the history of science itself. A coincidence is a signpost that pops up to guide us, and is a vital part of creativity, and an acknowledged part of the Perennial Philosophy.

Physics contains many beautiful things that are walled off from most of the population by the language barrier of mathematics. There are many good popular science books that try to explain physics without using mathematics, but some of the most profound subjects are still secrets known only to physicists. In this book I have tried to give some insight into these hidden treasures of physics, using as little mathematics as possible, and explaining as I go — nothing requiring knowledge beyond school level. For simplicity however, because there are so many huge numbers involved, I use the standard scientific notation for powers of ten, writing a million for example as 6 powers of 10, 10^6, or one millionth as 10^{-6}. Trying to write a number with 51 powers of ten, 10^{51}, in words is confusing and ridiculous. The large numbers in this book are rounded to the nearest power of ten, this being common among physicists when discussing general ideas.

As well as showing some of the fundamental principles of computers and information physics, I cover the Principle of Least Action in Chapter 6, and Fourier Analysis in Chapter 10, both of which are jewels of mathematics and physics, and vital to understanding the arguments of this book. The Principle of Least Action asserts that Nature does everything in *the most perfect way possible*, and can be used to derive many of the most important theories in physics, including Newtonian mechanics, general relativity, and quantum mechanics. It is not generally taught to physicists until post-graduate level. Fourier Analysis is widely used throughout science and engineering and allows information to be represented in dual domains, the representations appearing to be quite different, yet each containing *all* the same information. What is localised in one domain is global in the other, and vice versa. Information that changes in time in one domain is eternal in the other. These subjects are extraordinarily deep and interesting, and I hope to give some understanding of how they work, and how they fit in with the Perennial Philosophy.

Many books and papers on physics have helped me, and some that have been indispensible are picked out in the bibliography at the end of this book. Along the way, I have again and again been impressed by the way my vision of the world as ultimate computer fitted so well with known physics, and offered new insights into many of its profound mysteries. The *Feynman Lectures on Computation* and *Feynman and Computation* have been the most eye-opening guides to information physics, and *The Feynman Lectures* (which are also available online) are the best text books I have ever read — such depth and such simplicity. I would never have believed that reading a book on physics could be as pleasurable as reading a novel.

And I found that I was not alone in seeing the Universe as a giant computer. The same perspective was held by several physicists including Konrad Zuse, creator in the 1930s of the first digital computers based on relays, and Ed Fredkin, MIT professor and pioneer of reversible computing, who also believes that the Universe is performing a single calculation. Richard Feynman was deeply interested in computation and the physics of information. Before electronic computers were available, he had organised human computers at Los Alamos to perform the complex calculations required to design the first atomic weapons, and he later played a central role in promoting the study of the physics of information, devoting much of the last 10 years of his life to this. He is also credited with suggesting the idea of quantum computation. Feynman's PhD mentor, John Wheeler, coined the slogan *It from Bit* as a rallying slogan to younger physicists, expressing his own conviction that physics is, at its deepest level, all about information, and that reality arises from processing this information. In recent years the idea that we live in a computer simulation of reality has become popular, but nobody can say what sort of base reality might host this simulation — if the most fundamental layer of physics is information processing, how can there be a base reality? The Perennial Philosophy says that, underlying our reality is the transcendental, timeless One that created, and maintains our real Universe.

Most people think that computers are complicated. This is true if you look at a complete computer, or even a silicon chip, but fundamentally, computing is surprisingly simple, you just need some physical quantities that can represent two possible states, 0 and 1, and that can be arranged to interact in a few simple ways. A useful computer contains very large numbers of individually simple logic gates. We will look at some of the great variety of types of computer in Chapter 9, and also see how ubiquitous information processing is.

In 1998 physicists Norman Margolus and Lev Levitin published a ground breaking paper that gives the precise equivalence between mass-energy and rate of computation. The Margolus-Levitin theorem allows Einstein's famous equation to be extended to the form $E = mc^2$ = logical operations per second, and has allowed major advances in understanding, including Seth Lloyd's description of the Universe as computer in his book *Programming the Universe*. The exchange rate set by Margolus-Levitin is 10^{51} ops/sec per kilogram, which means that a single hydrogen atom weighing just 10^{-27} kg, performs 10^{24} operations per second, considerably more than all the computing devices built by humans so far working together. Imagine the power in a grain of sand that weights 10^{20} more than an atom, it makes your smart-phone appear really puny. Perhaps William Blake had a sense of this underlying power when he wrote his famous lines:

> To see a World in a Grain of Sand
> And a Heaven in a Wild Flower
> Hold Infinity in the palm of your hand
> And Eternity in an hour

Why does an atom need such computing power? In fact, being proportional to mass, most of the power of this natural information technology resides in the nucleus at a scale 10^5 (100,000) times smaller than the atom itself. What is all this hidden, internal processing doing? Lloyd suggests that the Universe simply computes itself, but this surely cannot be the whole story. An atom seems to play a fairly modest role in our world, so why would it need all this computing power concealed inside it? I will argue that atoms not only form material matter, and launch and catch photons of light, but also act as information routers, handling communications between dual, inner and outer information domains of the Universe. Atoms separate these dual domains, or worlds, in the scale dimension, and maintain the *as above so below* duality of information in the Universe. The dimension of pure scale plays a very special role in the Universe which we have so far failed to understand. The Scale Dimension will be looked at in some detail in Chapter 12.

Symmetry became one of the most important concepts in 20th century physics after mathematician Emmy Noether published her 1918 theorems showing that every conservation law such as conservation of energy and momentum, also expresses a symmetry. Duality is a particular type of symmetry that has been described by mathematical physicist Shahn Majid as the deepest meta-principle in physics. There are many examples of duality in physics, just as there are in the Perennial Philosophy, and some will be considered in Chapter 5.

Duality of representation of information in the Universe is implied by the *Holographic Principle* which emerged from studies of the physics of black holes. Jacob Bekenstein and Stephen Hawking's discovery that the surface of a black hole is made up of maximally compressed information or entropy was developed by Gerard t'Hooft and Leonard Susskind into the principle that all the information in any 3D region of space also has a dual representation on the region's boundary surface. This boundary surface may not be visible, but the mathematics of general relativity implies its presence in some virtual form. The holographic principle tells us that the Universe as a whole has 10^{122} bits of information written on its 2D boundary surface, and this is a dual representation of all the 3D information inside.

So much for the outer boundary, but what about an inner boundary? I will argue that the Universe shares a common inner origin or focal point at the Planck scale of 10^{-35} metre. This may seem ridiculous, but there are many justifications that I will offer later in this book. Here are a few for starters:

1. There is plenty of evidence in physics that space, as something you can mark a location in, does not exist below the nuclear scale of 10^{-15} metre, and there are no permanently existing mass-carrying particles below that scale that could act as location markers anyway.

2. The 'space' at the nuclear scale of 10^{-15} metre is already 10^{15}, one million billion, times denser in mass-energy than ordinary matter made of atoms. This is not space as we know it, and it does not just exist inside an atomic nucleus, you need this extreme energy density to probe anything at all at these scales, hence our vast particle colliders like the LHC.

3. The big bang origin of the Universe has no location in space, one place is as good as another, why not at the centre of each mass-carrying particle? In fact, that is one way of thinking about the big bang, and it effectively privileges the dimension of scale.

4. Feynman specifically points out in his *Lectures on Physics* that while you can move an experiment at different speeds, and to different places, and get the same results, this is not true for changes of scale — the scale dimension is quite unlike the usual three dimensions of space.

5. Quantum mechanical *entanglement* provides an instantaneous connection between particles that are separated in space. Many physicists believe that every particle in the Universe is entangled. This makes sense if there is a shortcut across space through a common centre at the smallest scale. As

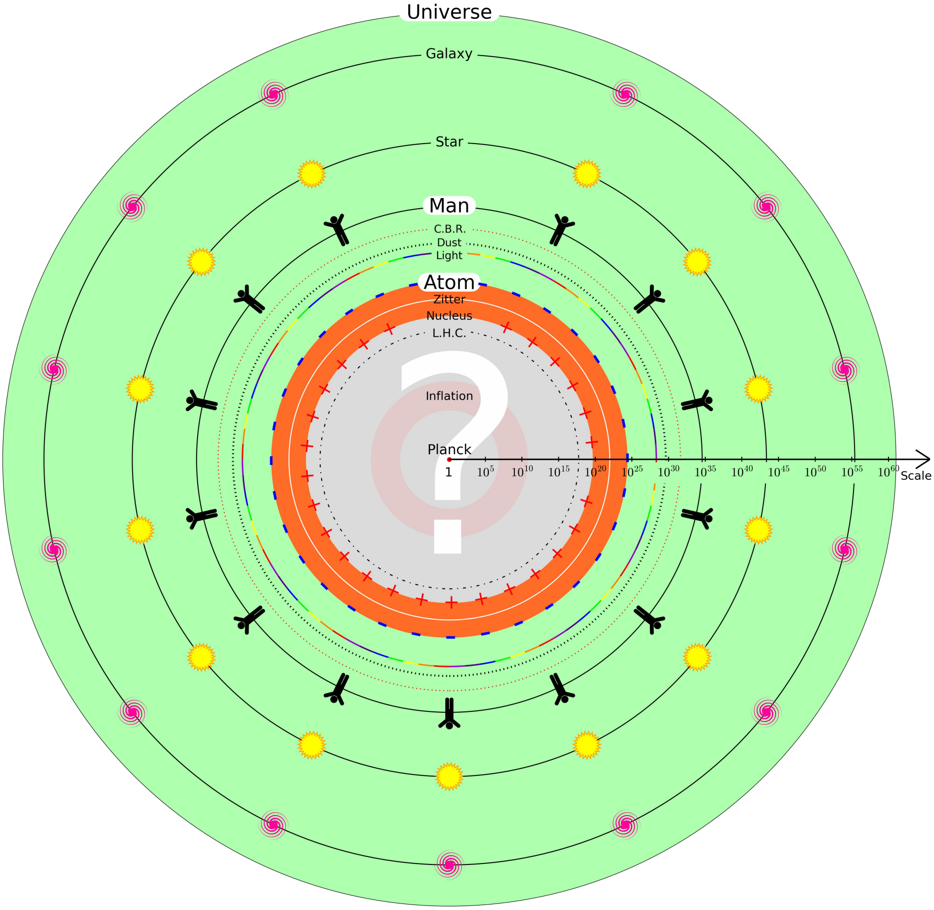
physics only recognises two forces that operate above the nuclear scale, electro-magnetism and gravity, how else could this trick be done?

6. The Upanishads and the Bhagavad Gita are some of the best expositions of the Perennial Philosophy, the distilled consensus of all mystic and religious experience, and their most repeated assertion is that the One who rules the Universe is located at the smallest scale *within* everything.

I also believe that more ‘primitive’ human cultures would have had no problem with the concept of an inner connection within each particle to the One at the centre of creation; it is our presence in an apparently 3D environment, and our separatist, unspiritual world that has led us astray. Even if the Planck Scale cannot be accurately described as a central location in normal space, this is a very suggestive way of looking at the Universe, and it is in accordance with the holographic principle.

The diagram below emerged after I had tried various other ways of representing the range of scales of the Universe. I later realised that it is a form of *mandala*, a picture that is designed to reorganise our consciousness in a particular way, relating Atom, Man and the Universe, so I call it an AUM mandala.

> AUM stands for the supreme Reality
> It is a symbol for what was, what is,
> And what shall be. AUM represents also
> What lies beyond past, present, and future.
> The *Mandukya Upanishad*, translated by Eknath Easwarran



Atom-Universe-Man AUM mandala

The range of scales of the Universe

The scales of the Universe span 61 powers of ten, 10^{61}, and are represented as a sort of cross-section through a *single* atom, with multiple people, stars, galaxies, etc, at larger scales.

The Planck scale of the central point at 10^{-35} metre is the smallest scale. Physics breaks down here, the temperature is about 10^{32} degrees, and the energy density so high that general relativity says it must form a black hole. You could also regard the Planck scale as the origin for the big bang, with the band marked inflation showing the brief period of exponential expansion required to make the big bang theory work. The outer circle, 10^{61} times bigger than the Planck scale, is at 10^{26} metres, the furthest observable distance, where general relativity again predicts that the Universe must close off as a black hole, or at least be very close to doing so —

the scale dimension ends in a black hole whether you go up or down! The dotted circle representing the size of dust is at the midpoint in scale, and is sandwiched between the Cosmic Background Radiation and the spectrum of visible light. This middle range of scale is the Goldilocks zone of the Universe. Notice also how the electric charges in atomic matter are separated into positive at the nuclear scale, with negative charges orbiting at the atomic scale. The intense electrical attractions between positive and negative charges bind each atom together and, repeated across 10^{78} or so atoms, this also holds the inner and outer domains of the Universe together. We will look at this diagram in much more detail in Chapter 12.

The laws of quantum mechanics that govern the inner small scale world are quite distinct from the more familiar laws that govern the macroscopic outer world at larger scales. Physicists use the completely different tools of quantum mechanics and general relativity to describe the small scale inner, and large scale outer worlds, and have tried without success for 90 years to join these theories together. Despite this, both theories can be derived from the idea of maximising perfection, using the Principle of Least Action that we look at in Chapter 6.

In the quantum world *every possibility is present at all times*, and quantum states evolve in ways precisely defined by quantum mechanics, it is only when the quantum world talks to the outer world, in what physicists call a measurement, that all possibilities are eliminated except for the actual outcome that we observe. Mainstream physics sees the choice of outcome from the range of possibilities as a completely random roll of the dice, unconditioned by anything. But Einstein hated this abandonment of cause and effect, and David Bohm showed later that the mathematics of quantum theory does allow the choice of outcome to be determined by a *consensus of the whole Universe*, through what he called a *quantum potential*. Bohm was largely ignored by other physicists as his ideas added extra structure to quantum mechanics without offering any benefits in the predictive power of calculations. Even if quantum mechanics is incomplete, as Einstein believed, it is supremely accurate in its predictions, and it underlies almost all modern technology.

The accelerating advance of technology since the millenium has allowed ever greater levels of surveillance by governments and corporations. This monitoring of our smallest actions is reminiscent of the traditional religious belief in a God who knows everything, down to our every thought. It seems logical that a centralised consciousness of the Universe, which for want of a better name I call God, would make use of similar, but vastly superior I.T. for surveillance and other functions such as providence, and karma enforcement. If every detail of your life is spied upon by Facebook, or Google or the CIA, why would God not use similar surveillance methods? The Chinese government is currently leading the world in the development of manmade karmic control systems, watching, scoring, and rewarding or punishing its citizens according to their behaviour, and their obedience to the ruling elite.

Karma — as you sow so shall you reap — is a common element of all religions even though they disagree about when the karmic consequences of an action will rebound on the actor. Soon after? Upon death? In a later lifetime? On Judgement day? Disagreement occurs even within a single religion such as Buddhism. If karma of some sort is at work in reality, how could it be enforced without some very advanced I.T.?

But what if you don't believe in God? Einstein was not religious, but frequently referred to God — God does not play dice, the Lord is subtle but not malicious, etc — but Einstein's idea of God was more like Spinoza's concept of God as Nature. The thesis of this book is that the outcome of every local event in the Universe is decided after consultation with the entire Universe by means of Nature's extraordinarily potent information technology. We may not individually like the outcome of each event, but that does not mean that it may not be the maximally perfect choice for the Universe as a whole, just as Newton's rival, and exponent of the Perennial Philosophy, Wilhelm Leibniz believed.

Although church attendance has declined dramatically in western countries, the majority of people still believe that everything does not happen by pure chance — there is something more. An atheist's belief that there is no God is no less a fixed unverifiable standpoint than that of a believer, it is faith in randomness. Software engineers increasingly use processes based on Darwinian evolution to write software that is too complex for humans to understand, harnessing the power of random mutations. But randomness can be deceptive, you can never be sure that a random appearance is not hiding some deeper level of organisation, and perfect randomness is unachievable by any computer. Even the random mutations of evolution in Nature are not acausal, they are the result of complex interactions of vast numbers of separate entities, apparently random, but not ultimately acausal. We will look at randomness in Chapter 6.

In 2017, the AlphaZero AI computer took just four hours to teach itself to play chess better than any human player, having been programmed with only the bare rules of the game. What goes on inside an AI machine like this is way beyond human understanding, our limited human brains cannot assimilate such complex information, and there are no concepts accessible to humans. It all appears random, but is very evidently not so, the AI is applying more intelligence to its task than any human could. Human chess and Go players now study the games played by such machines to learn new lines of play that have eluded human players for thousands of years — computers have become creative. In 2020, an AI fighter pilot shot down a human fighter pilot in five out of five simulated dogfights.

The prominent atheist Richard Dawkins is justified in his criticism of what he calls the *sky fairy* picture of God and the other numerous absurdities purveyed by the major religions to the gullible, and he is to be congratulated for his bravery in exposing fake truths. Most people discontinue believing in Father Christmas at around the age of seven, as the free imagination of childhood gives way to adult rationality. But for some reason people continue to believe unnecessarily in all manner of absurdities presented as facts by the main religions. Much of the bible makes no logical sense even though it is considered the foundation document of truth by Christians, Muslims and Jews. The young Einstein was drawn to religious belief in God, but at around the age of twelve he became disgusted by religious falsehoods and the necessity of believing in the unbelievable, and it was Euclid's *Elements* that became his holy book, its purely logical derivations of truths of geometry encouraged him to redirect his spiritual energy, and love of truth, into mathematics and science. In the west, very few politicians have scientific qualifications; perhaps this is one reason why they find it so easy to spout untruths.

Refusal to accept a dice playing God was one of the main reasons that Einstein never accepted that quantum mechanics is a complete theory, despite having been the first physicist to take seriously the quantum concept that Max Planck had reluctantly introduced in 1900. It was pursuit of the logical consequences of the quantum idea beyond the emission of light that enabled Einstein to produce many of his great early papers, including his 1905 explanation of the photoelectric effect that later won him the Nobel prize.

Eastern religions including Buddhism and Hinduism/yoga have no room for a dice playing god either, and are therefore more logical than 20th century physics — all is cause and effect, there is no place for acausal behaviour. Buddha said that his D*ependent Origination*, with its 12 steps that Buddhist monks still chant 2500 years later, was his deepest teaching, and that it applies to every process in Nature.

It is fundamental in science that all humans are fallible, and the same applies to all texts written by fallible humans. Einstein blundered many times — *and admitted it*. Maxwell was notorious amongst his contemporaries for errors in his mathematics, but that has not held back his status as one of the three greatest physicists in history. Ultimately, in physics, errors are exposed by conflict with experimental evidence, and the criticism of other physicists. Physicists accept what Feynman called *the primacy of doubt*, it may be painful for egos, but is essential to uncovering truth. If only more politicians and religious leaders were so reasonable and honest…

I have tried to ensure that the physics in this book is correct, but there may be errors for which I apologise. You can send corrections or comments to:

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The interpretations of physicists' work that I make are entirely my own and should not be attributed to anyone quoted.

1
Science and Spirit

Lack of Progress in Theoretical Physics

Despite the ever increasing number of physicists and research programmes, fundamental physics has made little progress for years. The discovery of the Higgs boson by the LHC in 2012, and the observation of Gravitational waves by LIGO in 2015, were triumphs of experimental physics, but only confirmed theoretical predictions that were decades old. String theory has occupied much of the energy of a whole generation of physicists without producing a single experimentally testable prediction.

> For the past twenty-five years we have certainly been very busy. There has been enormous progress in applying established theories to diverse subjects: the properties of materials, the molecular physics underlying biology, the dynamics of vast clusters of stars. But when it comes to extending our knowledge of the laws of nature, we have made no real headway. Many beautiful ideas have been explored, and there have been remarkable particle-accelerator experiments and cosmological observations, but these have mainly served to confirm existing theory. There have been few leaps forward, and none as definitive or important as those of the previous two hundred years. When something like this happens in sports or business, it's called hitting the wall.
> Lee Smolin, *The Trouble with Physics*, 2008

Another physicist describes a lot of modern work as "fairytale physics"

> . . . modern science has discovered that the reality of our physical existence is bizarre in many ways, but this is bizarreness for which there is an accumulated body of accepted scientific evidence. There is as yet no observational evidence for many of the concepts of contemporary physics, such as supersymmetric particles, superstrings, the multiverse, the universe as information, the holographic principle or the anthropic cosmological principle. For some of the wilder speculations of the theorists there can by definition never be any such evidence.
>
> This stuff is not only not true, it is not even science. I call it 'fairy-tale physics'. It is arguably borderline confidence trickery.
> Jim Baggott, *Farewell to Reality*, 2013

While physicists can calculate the behaviour of all sorts of processes with impressive accuracy, there is a dire lack of understanding and insight into what any of it means, and this leaves a student of physics with a feeling of philosophical starvation. The 'shut up and calculate' Copenhagen interpretation of quantum mechanics still reigns.

You certainly have to think clearly and avoid the irrational, but at the same time it is foolish to shut out ideas that may lead to new layers of truth. The Perennial Philosophy, which we will look at in Chapter 2, provides a perspective on reality, distilled from the consensus of all human spiritual experience after editing out the irrational, the miracles, and any reliance on infallible persons or texts. At every stage in the history of mathematics and science there have been established physicists resisting the adoption of new ideas. A famous example is Max Planck, who, after spending twenty years criticising Ludwig Boltzmann's contributions to statistical mechanics, found himself forced to make use of Boltzmann's methods in order to derive the famous Planck radiation law that gave birth to the quantum. Having discovered the quantum, Planck disliked it so much that he tried for years to get rid of this minimum unit of energy that had appeared in his mathematics, but he failed. Planck was his own counter-revolutionary. It took the revolutionary young Einstein to pick up Planck's quantum and run with it, writing many of his greatest papers by exploring its consequences.

Two of the ideas of modern physics that Baggott criticises, the Universe as information, and the holographic principle, are essential to this book and its aim of connecting physics with the Perennial Philosophy. I disagree with Baggott about the lack of evidence for the Universe as information. Quite apart from my own experience described in the introduction, quantum mechanics is all about information, what we can know and what we cannot, and we are racing to harness the extraordinary power of quantum computing that is inherent in Nature at the atomic level. As nobody knows all the truth, we can only keep our minds open, trust our intuitions, and think for ourselves.

Gods of Physics

If you think that religion and spirituality have no relevance to physics consider this. Who were the greatest physicists in history? If you ask physicists for their opinion, most will top their lists with Newton, Maxwell and Einstein.

Isaac Newton, a devout Christian, spent more of his time on theology and alchemy than on physics.

James Clerk Maxwell formulated the famous set of equations bearing his name that give a complete mathematical description of electrical and magnetic phenomena and also showed that light is an electromagnetic wave. Maxwell’s equations not only revealed the possibility of wireless communication using electromagnetic waves, but even turned out to contain the Lorentz transformations of relativity encoded within them. Maxwell also pioneered many other areas of physics that are now fundamental to the subject, introducing field theory and statistical mechanics. He developed colour theory and took the first colour photograph. He also founded the Cavendish laboratory.

> From a long view of the history of mankind — seen from, say, ten thousand years from now — there can be little doubt that the most significant event of the nineteenth century will be judged as Maxwell's discovery of the laws of electrodynamics.
> Richard Feynman, *The Feynman Lectures on Physics*, vol 2

> It is sometimes said, with no more than slight overstatement, that if you trace every line of modern physical research to its starting point you come back to Maxwell. Professor C. A. Coulson put it another way: 'There is scarcely a single topic that he touched upon which he did not change almost beyond recognition'.
> Basil Mahon, *The Man Who Changed Everything*

Maxwell was, like Newton, a committed Christian given to deep self-examination. He felt that something deeper than himself was the source of his creativity:

> . . . what is done by what I call myself is, I feel, done by something greater than myself in me.
> Maxwell speaking on his deathbed to Rev. Prof. Holt

The following lines from one of his poems seem particularly in tune with the Perennial Philosophy:

> There are powers and thoughts within us, that we know not till they rise
> Through the stream of conscious action from where Self in secret lies.
> But where will and sense are silent, by the thoughts that come and go
> We may trace the rocks and eddies in the hidden depths below.
> James Clerk Maxwell, *Recollections of Dreamland*, (Lines 48-51)

Einstein was not religious in the conventional manner, but had such faith in the supreme intelligence of the creator of the Universe that he used this faith to guide his unmatched intuition. After publishing what became known as special relativity in 1905, he worked for 10 years to include gravity in relativity theory, starting with little but a sense of how things ought to be. Most discoveries in physics have not been made this way. Einstein's recorded remarks reveal something of his mode of thought.

> Subtle is the Lord. Malicious, He is not.
> Albert Einstein

Scientists always need to believe that the processes of Nature are open to human investigation and understanding. Einstein seems to have believed that the workings of God are also accessible.

> God is a mystery, but a comprehensible mystery.
> Albert Einstein

In this book, I argue that our growing understanding of information technology and its role in fundamental physics will allow us to understand the architecture of God's mind in terms of I.T. It does not really matter what the word God signifies to you, it may be one of the names for the Almighty One used by the main religions, or just the Cosmic Computer, but even if you attribute all power to randomness, that God also has remarkable powers. We will look at some of these in Chapter 2.

In his book *The Perennial Philosophy,* Aldous Huxley used the scientific principles of fallibility and consensus as filters for religious ideas. Einstein went further in allowing that not only could influences go in either direction, but that this process is of mutual benefit to both science and religion.

> Science without religion is lame, religion without science is blind.
> Albert Einstein

Huxley showed how scientific thinking could relieve religion of its lameness. Can The Perennial Philosophy help physicists to find something important they have been missing?

Eastern Ideas

During the troubled birth of quantum mechanics in the 1920s, mysteries such as the wave-particle duality discovered by Louis de Broglie, suggested to some physicists that they might need to incorporate Eastern ideas into their Western thinking to help understand such apparent contradictions. Niels Bohr's Copenhagen interpretation of quantum mechanics, which became dominant in 1927, put an end to this, attributing the choice of quantum measurement outcomes to the God of randomness, and sidelining the more holistic ideas of de Broglie, Schrödinger and Einstein, who thought that deeper processes must be at work. Bohr had effectively put up a wall on the frontier of knowledge, saying ‘look no further, there is nothing meaningful to be found out there’, and it became a career threatening option for physicists who dared to explore further. Quantum mechanics was such a successful system of calculation, that many physicists preferred not to worry about deeper questions. When David Bohm showed later that quantum mechanics could be made holistic by the small addition of a quantum potential, he was widely ignored, as his ideas made no visible change to the probabilistic outcomes of calculations. Bohm was a friend of the Indian spiritual figure Jiddu Krishnamurti, and was well schooled in the Perennial Philosophy. He interpreted the processes of physics as a ‘holomovement’, a continual enfolding and unfolding between two domains that he called the implicate order and the explicate order. We will look further at Bohm’s ideas in Chapter 11.

Two assertions of mainstream physics are especially at odds with the Perennial Philosophy. First there is the denial of a universal Now which we will look at in Chapter 8. Second is the acceptance of unconditioned randomness. This abandonment of Leibniz’s principle of sufficient reason, which asserts that every effect must have a cause, is discussed in Chapter 6.

Paradoxes of Physics

Despite being built from logical theories, physics has many mysteries, and several paradoxes — contradictions in, or between theories, indicating that some assumption must be wrong. Prominent among these is the singularity problem. Physicists like to describe a particle as an infinitely small point, and while this model has been enormously fruitful, it has several pitfalls. The equations that describe the strength of the electric and gravitational fields produced by a particle require you to divide by the distance from its centre. At the particle’s actual location, the distance from its centre is zero, so you must divide by zero, but a computer will refuse to do this impossible task by stopping with a division by zero error. Even at a tiny distance from the centre of the particle, the answers you get will approach infinity, leading to the absurd prediction of electromagnetic theory that the energy of a point charge is infinite. It was ongoing problems of this sort that motivated the creation of string theory which avoids the problem by using tiny one dimensional strings rather than zero dimensional point particles.

The singularity problem also causes headaches for gravity theorists studying black holes. General relativity seems to work beautifully until you approach the singularity at the big bang, or the middle of a black hole, where theory breaks down with the need to divide by zero.

The 20^{th} century's two most successful theories were general relativity and quantum mechanics, and each has passed all experimental tests, and continues to reign in its own domain; GR at large scales, and QM at small scales. However, despite more than 50 years of work, physicists have been unable to join the two together, they just appear to be completely incompatible. A theory of quantum gravity is number one on the agenda of theoretical physics.

Quantum theory predicts that there are fluctuations in space-time going right down to the Planck scale of 10^{-35} metre. The energy contained in a sphere at the Planck scale would be the Planck energy, which is equivalent to about half a ton of TNT exploding. The observable Universe is 10^{61} times bigger than the Planck scale, and 10^{183} times the volume of a Planck sized sphere, so the total energy of all these fluctuations across the whole Universe, adds up to 10^{183} times the Planck energy. The trouble is that the Universe only contains a total of about 10^{61} times the Planck energy. The discrepancy factor of 10^{122} between these two has been described by physicists as "the worst theoretical prediction in the history of physics".

The ideas presented in this book shed new light on these and many other mysteries of physics, making sense of what has appeared baffling, especially quantum mechanics. The central idea is that the world is divided in the dimension of pure scale between dual inner and outer domains, and that the information in each domain mirrors the information in the other but is encoded differently. The two domains are in constant communication with each other through atoms which provide the gateways between these inner and outer worlds. This is more or less a rephrasing of David Bohm's concept of a holomovement that continually enfolds and unfolds to create reality. But we can now take advantage of our knowledge of I.T. and the physics of information, to outline the architecture of the Universe as Cosmic Computer.

Particles

It is difficult to pin down a fixed scale that marks the border between the inner and outer domains, as this is determined by the size of Planck's constant, (later referred to as a Planck), which is a composite of mass, length and time units. However, at about the atomic scale and below, physicists have to change their toolkit completely, abandoning Newton's outer world mechanics in favour of the mysterious, but spectacularly accurate, quantum mechanics, which constructs everything around the size of a Planck. Placing the border at the scale of an atom is not strictly correct as quantum effects are noticeable at larger scales too, but this border is permeable like any other, and it is a convenient simplification supported by the fact that atoms and their constituent particles are the principle functional hardware of the Cosmic Computer that is our world, as we will now see.

Physicists have discovered hundreds of what they call particles, but the only ones that have lifetimes longer than a microsecond, 10^{-6} sec, and contain rest mass that allows them to stand still rather than travel at light speed, are the neutrons, electrons, and protons that make up atoms. Even among these three, a neutron only exists for an average of 13 minutes before decaying, unless it is stabilised by confinement inside an atomic nucleus or a neutron star. Electrons survive for at least 10^{28} years, and protons 10^{33} years, each comfortably outliving the Universe, which is only 10^{14} years old. The recently 'discovered' Higgs particle has a lifetime of 10^{-22} second, so it would have to live 10^{22} times longer to last for a single human heartbeat. Other particles like quarks cannot exist independently at all, the protons and neutrons of an atomic nucleus cannot be pulled apart to reveal quarks, because the nuclear scale strong force gets stronger with distance, unlike the outer world forces of electro-magnetism and gravity which get weaker with distance.

Atoms are the only source of light in the Universe. Photons, the quantum particles of light, are only emitted and absorbed by electrons in atomic orbits. A free electron, not bound in an atom, is unable to do this, it is too light to absorb the recoil without the stability provided by the much greater mass of an atom's nucleus.

The idea that atoms and their constituent particles act as communication nodes between inner and outer domains of the Universe suggests many new lines of thought. Thinking like an engineer rather than a physicist, it is natural to try and trace how information passes from one level of scale in the Universe to another, and how all these levels are kept in synch, especially the 'conjugate' quantities of quantum mechanics like energy and time, and position and momentum. The two components of a conjugate pair are Fourier duals of each other, one component can be transformed into the other by a mathematical technique called Fourier analysis which we will look at in Chapter 10. Every smartphone has a DSP (Digital Signal Processing) chip, devoted mainly to the task of performing Fourier transforms, and so ubiquitous is the use of Fourier transforms in science and engineering that more DSP chips are manufactured than CPUs (Central Processing Units). The fact that the mathematical processes going on in the equations of quantum mechanics are just the same as those we use as mainstays of I.T. is surely a strong clue to the importance of information processing in physics. We look at Fourier duality and Fourier transforms in Chapter 10.

In Chapter 5 we see how the usefulness of floating point 'real' numbers and infinitesimal calculus has beguiled us about the true nature of numbers. There have been many objections by mathematicians over the centuries as new forms of number going beyond the integers 1, 2, 3, … have emerged. In a computational Universe, some of these objections should be taken seriously. The debate over whether space is a continuum with no minimum scale, has been going on for more than 2000 years, and is still unsettled despite Max Planck's unveiling of his quantum more than 120 years ago. The evidence now seems overwhelming that the continuum is a convenient mathematical fiction, and we need to recognise this, and pay more attention to the scale level where the Planck digit reigns; this is the focus of Chapter 13.

Machine Code

As a working programmer in the 1980s I wrote a lot of code in assembler, a primitive programming language which just uses a small set of instructions like MOV a,b to move a number from memory location a to memory location b, but can do few higher level tasks, sometimes not even multiplication or division. Each assembler instruction translates directly into machine code, the numerical instructions that a microprocessor understands. This is as close to the hardware as a programmer can get. The advantage of using assembler over higher level languages like c or BASIC was speed. A program would often run hundreds of times faster when written in assembler, making it possible to do things that seemed impossible on early microprocessors, even to other programmers. When you wrote the code to divide one number by another it was essential to avoid dividing by zero. If this happened the computer crashed and you had to hit CTRL+ALT+DEL and wait for several minutes for it to restart, so you always needed to check if the divisor was zero, and if so, jump to special code you had written to handle it.

Special treatment of zero also features in the theory of matrices and linear transformations that are among the most important tools used by physicists. Each row or column of numbers in a matrix is a statement of rules connecting quantities. The determinant of a matrix is a number which shows the overall scaling effect of the matrix. The matrix is called *singular* if the determinant is zero, and this reveals that two rows, or two columns of the matrix are encoding the same information. The mathematics of taking the determinant seems to sniff out any duplication of information within the matrix and give it zero overall worth if there is. Matrices are looked at in more detail in Chapter 4.

There is another way in which my thinking was affected by years of writing programs that produced graphical output on a screen or printer — pixelation. Mathematicians and most physicists have no problem discussing very big or small quantities, but programmers know that the size of a pixel is crucial, and failing to allow for it can cause you many problems. From this programmer's viewpoint, it is surprising that physicists were so puzzled by the emergence of the quantum in 1900, and took so long to accept that all of Nature is quantised. Pixel, quantum, bit, they all mean much the same thing, the smallest viable unit of something.

The differential equations of the calculus created by Newton and Leibniz rely on infinitesimals, and are still standard for mathematics and physics. But engineers working in fields such as signal processing often use difference equations which work with fixed small sized steps rather than infinitesimals. Charles Babbage designed his difference engine to use small fixed sized quantities rather than infinitesimals — to operate in the real mechanical world that is what you have to do. Modern computers use 64 bit registers to store numbers, and clever mathematical tricks that enable them to give the illusion that they can store any number you want, but scientists who require high precision still have to use special techniques.

Another lesson I learned from programming in the days of 8-bit or 16-bit computers that lacked floating point arithmetic was the problem of register overflow. A register is a collection of memory bits to hold information. An 8-bit register can store any number from 0 to 255, a total of 2^8 possible values, and a 16-bit register can hold any number from 0 to 65535, 2^{16} possible values. If your 16-bit register holds 65534, you can add one and get 65535, so that is fine. But if you add one again, it goes to 0. A lot of extra code has to be written to avoid this type of problem. It is possible that the Cosmic Computer has a fixed maximum register size, and this might account for the repeated appearance in physics of numbers of about 10^{20} and multiples of this like 10^{40}, and 10^{80}. For example, the gravitational force is about 10^{40} times weaker than electro-magnetism, the nuclear scale is about 10^{20} times larger than the Planck scale, and there are around 10^{80} atoms in the Universe. So perhaps gravitation is a manifestation of register overflow. The coincidence of these large numbers has fascinated many physicists including Eddington and Dirac. Dirac set out his ideas in his 1937 *large numbers hypothesis*.

So if there is a fixed size of pixel, or quantum, and you want to go smaller, what then? Things get weird. Welcome to quantum mechanics, which is further discussed in Chapter 11. Atoms are the pixels of conventional matter, there is nothing smaller that you can assemble into a 3D object. The scale of an atom's nucleus may be yet more fundamental, the pixel size of space, and we will look at the reasons for this in Chapter 13.

2
The Perennial Philosophy

God or no God?

Is there some sort of God in charge of everything? And what do we mean by God anyway? Do we really have free will or are we mechanically following a pre-written script? What happens when we die? Without any clear answers to any of these questions, it is easiest to stick with whatever religion is standard in your culture. Many people do just this, through cultural pressure and brainwashing, ignorance, fear of God and priests, or through simply playing safe, taking out an insurance policy against ending up in hell. Given the luxury of freedom to choose, some profess an equally dogmatic faith in the no-God of atheism, and the middle ground is held by agnostics, the don't-knows. Faith is deemed essential by most religions, but if you do not already have it, why should you trust what the faithful tell you? The obvious solution for anyone capable of thinking for themselves is to adopt a scientific, evidence based approach, and trust your own judgement.

> Like the bee gathering honey from different flowers, the wise man accepts the essence of different Scriptures and sees only the good in all religions.
> From the *Srimad Bhagavatam*, quoted in *The Perennial Philosophy*

Atheism

In his book *The God Delusion*, Richard Dawkins demolishes the sloppy thinking of theologians and the nonsense that established religions expect their followers to swallow and hold as truth. He also shows the impossibility of constructing a system of morality from the contradictions, and often repulsive examples in the bible. As a leading scientist whose thinking is clear and logical, he criticises theologians he encountered at a Cambridge conference who maintained that God lies outside science, and that other ways of knowing, besides the scientific, must be deployed to know God.

> The most important of these other ways of knowing turned out to be personal, subjective experience of God. Several of the discussants at Cambridge claimed that God spoke to them, inside their heads, just as vividly and as personally as another human might. I have dealt with illusion and hallucination in Chapter 3, but at the Cambridge conference I added two points. First that if

> God really did communicate with humans that fact would emphatically not lie outside science. God comes bursting through from whatever other-worldly domain is his natural abode, crashing through into our world where his messages can be intercepted by human brains — and that phenomenon has nothing to do with science? Second, a God who is capable of sending intelligible signals to millions of people simultaneously, and of receiving messages from all of them simultaneously, cannot be, whatever else he might be, simple. Such bandwidth! God may not have a brain made of neurones, or a CPU made of silicon, but if he has the powers attributed to him he must have something far more elaborately and non-randomly constructed than the largest brain or the largest computer we know.
>
> Richard Dawkins, *The God Delusion*

Dawkins is saying in the first of his two points that if there is communication from God to man, science should ultimately be able to unravel the mechanism. I agree, and this is one purpose of the current work, because our growing understanding of computation and the physics of information is giving us the necessary tools to develop greater understanding. Dawkins is also right to doubt any individual's interpretation of 'communications' they receive in their minds. Even the greatest scientists make mistakes, and it is one of the great glories of science that this is understood, and only a consensus of opinions that can be verified by experiments is considered true. If only this gold standard would be applied by religious leaders.

Experiences in meditation are bound to be subjective, but it is still possible to construct a consensus from the experiences of a great number of meditators and mystics over the ages, and this is the Perennial Philosophy. Brain scanning technology has now given scientists a powerful new tool for investigating the extraordinary power of silence and meditation, and shown that olympic level meditators from Tibet can produce gamma waves in their brain scans at levels never before observed in humans. In their book *Altered Traits*, Daniel Goleman and Richard Davidson describe the progress in this field, and the evidence that shows how meditation can rewire our brains.

Dawkins' second point, the lack of bandwidth argument, was partly what made me reject my Christian faith as a child: how could God know everything and be present everywhere? Science seemed to leave no room for this; what communication channel could God use? It is central to this book that the new understanding of physics as information processing, and the mind boggling power of Nature’s I.T. that has been revealed, make this argument unsustainable. With more processing power in an atom than the sum total of human built I.T., and the instantaneous synchronisation of information performed by the mysterious quantum phenomenon of entanglement, anything seems possible. Science is rapidly developing the means to understand at least the architecture of the mind of God, even though God's detailed thoughts may be for ever inaccessible.

Dawkins justifies his atheism by the lack of hard evidence for the existence of God and his understandable contempt for the religious establishment. He became famous for his book *The Selfish Gene*, in which he argued that it is the evolution of genes in DNA that controls living processes, including humans. Darwinian evolution works by looping round the following steps:

Random mutations in the DNA of an organism affect its offspring.
Beneficial mutations survive better when tested against the environment.
Loop

Here again, just as in quantum mechanics, the scientist's God of randomness is at work. Evolution works very effectively, as we can see in the wonders of Nature, and it is now a very important tool for developing human technology. No longer do programmers have to struggle to write code to perform a task such as recognising whether or not there is a cat in a picture. For this task, you only need a large training data set of pictures that have been labelled yes or no by humans as to their cat content, and a neural network with initially arbitrary strengths for its internal connections. The network is shown all the training pictures and its accuracy recorded. The connection values are given small random mutations and the process repeated. If the mutation results in better accuracy of cat recognition it is kept, otherwise not. By repeating this process many times the network can recognise a cat picture as reliably as a human, but would be utterly useless at any other task. The strengths of the resulting internal connections of the network are meaningless to a human, there is nothing accessible to human understanding. As with the mind of God, the network's architecture may be understood, but not its thoughts.

There is no doubt that evolution works and random mutations can be harnessed to do things beyond the capability of humans; much of modern technology depends on these facts. But as I repeatedly stress, randomness can be misleading, and there may be deeper levels of complex logic underlying it. The indestructibility of information that is asserted by physics forbids the possibility of a perfect crime. Even if all forensic evidence of the crime was shredded and scattered as widely as possible, hidden as 'random' variations of the surroundings, Nature as a whole would still record traces of the truth.

The Perennial Philosophy

The Perennial Philosophy is a cumbersome name for the consensus of mystics from all cultures over thousands of years, that there is an eternal divine ground to Reality, and that it is accessible to humans who have fulfilled the necessary conditions. The individual self in each of us is connected to, or in some manner identical to, the eternal Self of the divine Ground. In his book *The Perennial Philosophy* published in 1945, Aldous Huxley assembled a brilliant collection of quotations that highlight the unity of vision among mystics from all religious traditions. Just as scientific knowledge is derived from a consensus of the understanding and experiences of many individual scientists, and is verifiable by experiment, the Perennial Philosophy is derived from a consensus of the understanding and experiences of many individual mystics, and is verifiable by anyone prepared to devote their time to meditation.

> . . . this teaching is expressed most succinctly in the Sanskrit formula, tat tvam asi; ("That art thou"); the Atman, or immanent eternal Self, is one with Brahman, the Absolute Principle of all existence; and the last end of every human being is to discover the fact for himself, to find out Who he really is."
>
> The ground in which the multifarious and time-bound psyche is rooted is a simple, timeless awareness. By making ourselves pure in heart and poor in spirit we can discover and be identified with this awareness. In the spirit we not only have, but are, the unitive knowledge of the divine Ground.
>
> The divine Ground of all existence is a spiritual Absolute, ineffable in terms of discursive thought, but (in certain circumstances) susceptible of being directly experienced and realized by the human being. This Absolute is the God-without-form of Hindu and Christian mystical phraseology. The last end of man, the ultimate reason for human existence, is unitive knowledge of the divine Ground — the knowledge that can come only to those who are prepared to "die to self" and so make room, as it were, for God. Out of any given generation of men and women very few will achieve the final end of human existence; but the opportunity for coming to unitive knowledge will, in one way or another, continually be offered until all sentient beings realize Who in fact they are.
> Aldous Huxley, *The Perennial Philosophy*

The Perennial Philosophy is not a sacred text, or a set of fixed rules. All humans are fallible and we have to make the best judgements we can while still allowing for this. If everyone would accept this fundamental principle of fallibility, much human misery would be averted. History is full of tyrants who thought they had all the answers, and there are plenty around today, but if we are fortunate enough to have the freedom to do so, we all have the power to think for ourselves and not blindly accept another's views.

Aldous Huxley was the grandson of Thomas Huxley, who was called Darwin's bulldog after his robust support for Darwin's theory of evolution. Aldous Huxley used scientific principles to separate the pure essence of all religious and mystic experience from superstition. Quoting great mystics and saints from all the main religions, he highlighted the common ground they have all shared while denying the necessity for belief in unverifiable miracles, or the infallibility of a particular person or text. Such forms of irrational, mental laziness, have even been condemned as counterproductive by some of the greatest spiritual masters — think for yourself, God gave you a brain, so use it! Huxley's approach relies on consensus among those most qualified to talk about spirituality and rejection of unverifiable superstitious beliefs.

> PHILOSOPHIA PERENNIS — the phrase was coined by Leibnitz; but the thing — the metaphysic that recognizes a divine Reality substantial to the world of things and lives and minds; the psychology that finds in the soul something similar to, or even identical with, divine Reality; the ethic that places man's final end in the knowledge of the immanent and transcendent Ground of all being — the thing is immemorial and universal. Rudiments of the Perennial Philosophy may be found among the traditionary lore of primitive peoples in every region of the world, and in its fully developed forms it has a place in every one of the higher religions. A version of this Highest Common Factor in all preceding and subsequent theologies was first committed to writing more than twenty-five centuries ago, and since that time the inexhaustible theme has been treated again and again, from the standpoint of every religious tradition and in all the principal languages of Asia and Europe.
> Aldous Huxley, *The Perennial Philosophy*

Huxley's account of The Perennial Philosophy became an important text for hippies and the New Age generation of the 1960s. The Beatles and many others went to India to absorb what Huxley had found to be some of the purest forms of the Perennial Philosophy. In his 1944 essay *The Minimum Working Hypothesis*, Huxley had already identified its key features:

> There is a Godhead or Ground, which is the unmanifested principle of all manifestation.
>
> The Ground is transcendent and immanent.
>
> It is possible for human beings to love, know and, from virtually, to become actually identified with the Ground.
>
> To achieve this unitive knowledge, to realise this supreme identity, is the final end and purpose of human existence.
>
> There is a Law or Dharma, which must be obeyed, a Tao or Way, which must be followed, if men are to achieve their final end.
> Aldous Huxley, *The Minimum Working Hypothesis*

The core message of the Perennial Philosophy that Huxley identifies is the Sanskrit formula *tat tvam asi* ("That art thou"). "The Atman, or immanent eternal Self, is one with Brahman, the Absolute Principle of all existence; and the last end of every human being is to discover the fact for himself, to find out Who he really is."

> The more God is in all things, the more He is outside them. The more He is within, the more without.
> Meister Eckhart (1260-1328), quoted in *The Perennial Philosophy*

This quotation from the christian mystic Meister Eckhart illustrates a major theme of the current book — the connectedness of everything in the Universe through the *Scale* dimension that physics has yet to acknowledge. The dictum of Hermes Trismegistus "As above, so below, as within, so without, as the Universe, so the soul" describes the duality between the peculiar inner microscopic world governed by quantum mechanics, and the familiar macroscopic outer world that is our home. There are many signs of this duality in modern physics.

Some critics of The Perennial Philosophy maintain that trying to marry up the contradictory viewpoints of different religions creates confusion, and that following the teachings of a single faith is the only path to God. While there is some truth in this, it was the failure to acknowledge the superior perspective of the Perennial Philosophy that has encouraged the endless religious wars which have scarred human history. Even within particular religions there are wars that would grieve their founders; Catholics fight Protestants, Sunnis fight Shias, etc. Followers of variants of the Perennial Philosophy that are present in all the religions, have often been persecuted for not staying on-side. Humans love to separate just about anything into the categories *them* and *us*, whether it is football teams, nations or religions.

Although you may make short term gains by following some narrow set of ideas, too tight an allegiance will ultimately hold you back.

> When a man follows the way of the world, or the way of the flesh, or the way of tradition (i.e. when he believes in religious rites and the letter of the scriptures, as though they were intrinsically sacred), knowledge of Reality cannot arise in him.
> Shankara, quoted in *The Perennial Philosophy*

It is quite remarkable how, despite mass 'higher' education, people are willing to accept what they are told without critical examination, even when it is plainly nonsense. Whereas an engineer would swiftly lose his job if an aeroplane he had designed could not fly, politicians responsible for disastrous policies seem to have little trouble getting reelected. Why is it that so few western politicians have any scientific training?

Most religious belief stems from ignorance as much as knowledge: you are required to believe in various unverifiable miracles, the infallibility of certain individuals, or the sacred perfection of some old and heavily edited text. This is all repulsive to someone who has learnt to think critically and weigh up evidence in a scientific manner. As children grow up they cease to believe in Father Christmas, but religious zealots seem to be able to continue to believe in equally fantastic magical phenomena. All religious texts have been created by humans, and edited and translated by more humans. There are only four gospels in the New Testament, inconvenient ones, like the Gnostic gospels, were left out. The satanic verses were removed from the Koran as 'influenced by satan', despite being dictated by Mohammed who is considered infallible. Etc, etc...

> When the Western world accepted Christianity, Caesar conquered; and the received text of western theology was edited by his lawyers.
> A.N.Whitehead, *Process and Reality*, Gifford Lectures, 1927-8

A Declaration of Intellectual Independence

It is evident that whatever is experienced in the highest mystic state of connection to the divine Ground is indescribable in words, but that does not mean that mystics are given to irrational thinking. They may access conscious states beyond thinking, but you could not find anyone more logical than, for example, Gautama Buddha. In a discourse to the Kalama people, the Kalama Sutta, he listed ten specific sources of knowledge which should not be accepted as truthful without further investigation:

1. Oral history
2. Tradition
3. News sources
4. Scriptures or other official texts
5. Suppositional reasoning
6. Philosophical dogmatism
7. Common sense
8. One’s own opinions
9. Experts
10. Authorities or one’s own teacher

This list might appear to leave no avenues open to the pursuit of knowledge, but the essence of Buddha's teachings was meditation with an open mind. Buddha's “declaration of intellectual independence”, as the Kalama Sutta has been called, is similar to Richard Feynman’s advocacy of the “primacy of doubt” in science. The main extra ingredient that modern science has adopted is verification by repeatable experiment, but even here, Buddha's recommendation was to try meditation for yourself in order to verify his teachings. Over thousands of years, the Perennial Philosophy has been verified by the repeated self-experimental findings of meditators, and recent research with brain scanners is revealing the extraordinary and beneficial effects of meditation practice. Of course, the last thing that most established religions want is for anyone to “go it alone”, as that would threaten and undermine their worldly power, and the livelihood of the priesthood.

Huxley is just as firm in rejecting sloppy thinking as Dawkins or Buddha:

> . . . so long as the Perennial Philosophy is accepted in its essential simplicity, there is no need of willed assent to propositions known in advance to be unverifiable.
> Aldous Huxley, *The Perennial Philosophy*

Clear, logical thinking is important, but so also is the different mode of approaching truth through the inner microscope of meditation.

> . . . the inspired rishis (literally "seers") of ancient India analysed their awareness of human experience to see if there was anything in it that was absolute. Their findings can be summarized in three statements which Aldous Huxley, following Leibnitz, has called the Perennial Philosophy because they appear in every age and civilization: (1) there is an infinite, changeless reality beneath the world of change; (2) this same reality lies at the core of every human personality; (3) the purpose of life is to discover this reality experientially: that is to realize God while here on earth. These principles and the interior experiments for realizing them were taught systematically in "forest academies" or ashrams — a tradition which continues unbroken after some three thousand years . . . The discoveries were systematically committed to memory (and eventually to writing) in the Upanishads, visionary documents that are the earliest and purest statement of the Perennial Philosophy.
> Eknath Easwaran, introduction to the *Bhagavad Gita*

Yoga

Although it is mostly seen in the west as a system of physical exercises, yoga is one of the purest forms of the Perennial Philosophy. There are innumerable paths of yoga, but all share the same goal. In India, Bhakti yoga, devotion to a personal deity, is the most followed. Hatha yoga is concerned with purification of the body and the *asana* postures that westerners think of as yoga. In Jnana yoga, the mind inquires into its own nature to transcend its identification with its thoughts and ego.

> Yoga can be said to constitute the very essence of the spirituality of India. The word yoga is derived from the root yuj, which means to unite or join together, much like the etymological meaning of the word 'religion'. The practice of yoga may lead to the union of the human and the divine — all within the self. It is a way to wholeness and to an integration of all aspects and levels of oneself. Yoga is not only a collection of certain practices devoid of a metaphysical basis. In fact, yoga is based upon a perfectly structured and integrated world view which aims at the transformation of a human being from his actual and unrefined form to a perfected form.
> Ravi Ravindra, *Yoga and the Teachings of Krishna*

Around 2000 years ago, Patanjali formulated his Yoga Sutras, a collection of short statements to guide the yogi. He starts by defining yoga:

> Yoga is the ability to direct the mind exclusively towards an object and sustain that direction without any distraction.
> Patanjali, *Yoga Sutras*, I-2

He never mentions postures (asanas) other than to say:

> Asana must have the dual qualities of alertness and relaxation.
> Patanjali, *Yoga Sutras*, II-46

The essential aim of yoga is to connect personally to the divine Ground and achieve liberation. The physical exercises, asanas, are just the third level on the eight runged (ashtanga) ladder leading up to the divine connection. Patanjali lists these in order:

1.	Yama	Moral control of behaviour
2.	Niyama	Purification of the body
3.	Asana	Body exercises (the well known postures)
4.	Pranayama	Breathing exercises
5.	Pratyahara	Restraint of the senses
6.	Dharana	Concentration
7.	Dhyana	Meditation
8.	Samadhi	Complete integration with the object of meditation

The first four of these are preparation of the mind and body for the later stages of meditation in which the consciousness is disconnected from the body's sensory inputs and all worldly concerns, ready for the very different state of consciousness of Samadhi. There are many levels of states of concentration which are known as the *jnanas* to Buddhists, and each has its own characteristics, but even if you only reach the lower levels it can be a life changing experience, an unforgettable sample of the "peace that passeth all understanding", the unconditioned bliss and happiness of eternity. Having experienced this, what fear can death inspire? Deeper levels allow the meditator to approach the centre of our consciousness.

> The mind, which is subject to change, and the Perceiver, which is not, are in proximity but are of distinct and different characters. When the mind is directed externally and acts mechanically towards an object there is either pleasure or pain. However when, at the appropriate time, an individual begins enquiry into the very nature of the link between the Perceiver and perception, the mind is disconnected from external objects and there arises the understanding of the Perceiver itself.
> Patanjali, *Yoga Sutras*, III-35

> When the mind is not linked to external objects and it does not reflect an external form to the Perceiver, then it takes the form of the Perceiver itself.
> Patanjali, *Yoga Sutras*, IV-22

> Thus the mind serves a dual purpose. It serves the Perceiver by representing the external to it. It also reflects or presents the Perceiver to itself for its own enlightenment.
> Patanjali, *Yoga Sutras*, IV-23

> Even though the mind has accumulated various impressions of different types it is always at the disposal of the Perceiver. This is because the mind cannot function without the power of the Perceiver.
> Patanjali, *Yoga Sutras*, IV-24

> A person of extraordinary clarity is one who is free from the desire to know the nature of the Perceiver.
> Patanjali, *Yoga Sutras*, IV-25

> And their clarity takes them to their only concern; to reach and remain in a state of freedom.
> Patanjali, *Yoga Sutras*, IV-26

Meditation

> The capacity to drive a thought away once and for all is the gateway to eternity. The infinite in an instant.
> Simone Weil, *Gravity and Grace*

Stopping yourself thinking is one of the most difficult things you can ever try to do. On a meditation retreat it generally takes two or three days of continual silence, meditating ten hours a day, to even approach the lower level states of concentration. Modern life, with all its stimulations and information overload, takes us ever further from these alternative, natural states of consciousness. Just as in the modern world, you need years of schooling and good exam results to enter university, in the time of Pythagoras, a solid practice of meditation was a prerequisite for entry into his school to study mathematics.

> Systematic training in recollection and meditation makes possible the mystical experience, which is a direct intuition of ultimate reality. . . This training is one which he will certainly find extremely tedious; for it involves, at first, the leading of a life of constant awareness and unremitting moral effort; second, steady practice in the technique of meditation, which is probably

> about as difficult as violin playing. But, however tedious, the training can be undertaken by anyone who wishes to do so. Those who have not undertaken the training can have no knowledge of the kind of experiences open to those who have undertaken it and are as little justified in denying the validity of those direct intuitions of an ultimate spiritual reality, at once transcendent and immanent, as were the Pisan professors who denied, on a priori grounds, the validity of Galileo's direct intuition (made possible by the telescope) of the fact that Jupiter has several moons. . .
> From *Beliefs*, an essay by Aldous Huxley

There seems to be a paradox in that, while we need critical thinking to avoid being duped and accepting whatever nonsense we are asked to believe, it is precisely when we stop thinking in silent meditation, that the door can open to deeper knowledge of Reality. To reach our full potential we require the best of both worlds, we need to find the perfect balance between our personal self and the Self that is identical with Brahman, the divine Ground. Huang-Po calls this the state of "no-mind"

> When followers of Zen fail to go beyond the world of their senses and thoughts, all their doings and movements are of no significance. But when the senses and thoughts are annihilated, all the passages to the Universal Mind are blocked, and no entrance then becomes possible. The original mind is to be recognized along with the workings of the senses and thoughts — only it does not belong to them, nor yet is it independent of them. Do not build up your views upon the senses and thoughts, do not base your understanding upon your senses and thoughts; but at the same time do not seek the Mind away from your senses and thoughts, do not try to grasp Reality by rejecting your senses and thoughts. When you are neither attached to, nor detached from them, then you will enjoy your perfect unobstructed freedom, then you have your seat of enlightenment.
> Huang-Po, quoted in *The Perennial Philosophy*

Drugs

Huxley tried many psychedelic drugs, even choosing to take mescaline when he was dying, and this was another part of his influence on the hippy generation. His book *The Doors of Perception* was required reading, its name quoting William Blake's lines:

> If the doors of perception were cleansed every thing would appear to man as it is, Infinite. For man has closed himself up, till he sees all things thro' narrow chinks of his cavern.

Huxley saw no separation between drug-induced and natural spiritual/mystic experiences. I should not have been surprised when I experienced effects during my first 10-day meditation retreat very similar to marijuana highs and the open hearted, loved-up states induced by the drug ecstasy; the doors of perception can be opened in many ways. Three days spent in meditative silence requires a lot more time and effort than toking a spliff, or popping a pill, but meditation is ultimately far more rewarding; shortcuts have their costs. Nowadays, you can also have a spiritual experience to order when a functional MRI machine targets a particular part of your brain.

Harvard psychologist Timothy Leary was greatly influenced by Huxley, and believed that anyone could have a spiritual experience after taking LSD, given the right set and setting. Leary and his colleague, Richard Alpert were fired from Harvard University in May 1963. Whilst later travelling in India, Alpert gave LSD to a guru, but the guru's serenity was unaffected, so Alpert gave him another tab of LSD. Still no effect. Alpert was sufficiently impressed to become a disciple of the guru, and changed his name to Ram Dass.

Psychoactive drugs and research on their properties was banned later in the 1960s and the war on drugs began and still continues… Governments were terrified by the prospect of people opening their doors of perception, and the dangerous ideas that were emerging: thinking for yourself, seeing through the lies of the ruling elite, opposing the war in Vietnam, promoting peace — far too dangerous. I can recall reading a book as a teenager in the 1960s arguing that it should be a basic human right to consume any God-created plant, provided that you did not harm anyone else. It also described taking an illegal drug as a victimless crime, and riding a horse as statistically far more dangerous to the individual than taking a banned drug. In 2009, Professor David Nutt wrote in an academic journal that the drug ecstasy was less dangerous than "equasy", a term he invented to describe people's addiction to horse-riding. For this plain truth he was sacked as the UK government's adviser on drugs.

A friend of mine at university in 1972 had a poster on his wall that had been issued by apartheid South Africa's police force. It warned in big letters: "MARIJUANA this is a very dangerous drug, under its influence people have been known to mix interracially without inhibition".

Humphry Davy was the famous chemist who invented the miner's safety lamp and mentored a young bookbinder with little formal education called Michael Faraday. Davy was a member of the Pneumatic Institution in Bristol, and socialised with the poets Samuel Coleridge and William Wordsworth. In a park in Bristol in 1798, he inhaled nitrous oxide from a silk balloon. When he recovered from its effects ten minutes later his first words were "Nothing exists but thoughts".

Once when I was travelling in India, I met a christian pastor who described to me the circumstances of his conversion. Some years earlier, in the depths of depression, he had decided to end his own life and taken an overdose of pills. A few hours later, rather than dying, Jesus came to him and changed his life — he was born again. I later recounted this story to my astute friend Alec who, after listening intently, surprised me by simply asking "What drug did he take?". Alec had tried all manner of psychoactive substances, and experienced many transcendental states, so he saw the pastor's tale from a different perspective. As Leary emphasised, set and setting is all important. The pastor, although not a believer before his experience, had grown up in a christian country, so it is hardly surprising that Jesus came to him rather than Krishna or Buddha or the prophet Mohammed. God may appear in many forms. Hinduism, which I have heard mocked by a Christian for having "50,000 gods", recognises the infinite diversity of forms in which the One may appear to men.

Another encounter I had on a train in India comes to mind. There was a man sitting opposite me with a gold bindhi mark on his forehead unlike the usual red bindhis often worn by women. I asked him what it signified and he told me the name of the Hindu sect that he belonged to. He then asked me what my religion was, so I told him I was a lapsed Christian. "Ah", he smiled and replied, "you are a devotee of Jesus". This way of describing a Christian struck me deeply, and revealed something of his way of thinking. The path of yoga most followed in India is *Bhakti* (devotion), and emphasises the intense mutual emotional attachment and love of a devotee toward a personal god, and of the god for the devotee. What god you choose is not ultimately important, each of the major divinities of Hinduism and the various forms of the Goddess have distinct devotional traditions. To my friend on the train, it did not matter that I came from a distinct devotional tradition, below the superficial differences, our paths must surely converge to the same destination.

> Howsoever men try to worship Me, so do I welcome them. By whatever path they travel, it leads to Me at last.
> *Bhagavad Gita* 4, 11

Mysticism

Mysticism might be cryptically described as a mode of life which claims, without long and laudatory praises of God, to bring us nearer to Him than do ordinary religious methods; as a view of life which rejects the all-too-human God made by man in his own image and out of his own imagination, replacing it by a formless divinity; and as a psychological technique which seeks to establish communication with this spirit, through the channel of interior contemplation.

Certain collective tenets of mysticism are not confined to any one faith, to any one country or to any one people, and are roughly universal. These cardinal positions of the mystic's thought are five in number and may be briefly picked out and exhibited as follows.

Mystics hold first that God is not to be located in any particular place, church or temple, but that His spirit is everywhere present in Nature and that Nature everywhere abides in it. The orthodox notion that God is a particular Person among many other persons, only much more powerful, yet still saddled with likes and dislikes, anger and jealousy, is rejected as childish. Pantheism is therefore the initial note to be sounded. Right thought hallows a place or makes it profane, and real sacredness dwells within the mind alone.

Next they hold that as a corollary from the first tenet God abides inside the heart of every man as the sun abides in all its myriad rays. He is not merely a physical body alone, as materialists believe, nor a body plus a ghost-like soul which emanates from it after death, as religionists believe, but He is here and now divine in the very flesh. The heavenly kingdom must be found whilst we are yet alive, or not at all. It is not a prize which is bestowed on us in the nebulous courts of death.

The practical consequence of this doctrine is embodied in the third tenet of the mystics, which asserts that it is perfectly possible for any man, who will submit to the prerequisite ascetic discipline, to enter into direct communion by contemplation and meditation with the spirit of God without the use of any priest or prelate as an intermediary and without the formal utterance of verbal prayer. This renders it quite unnecessary to lift upturned palms in suppliant adjuration of a higher Being. Silent aspiration thus replaces mechanical recitation.

The fourth tenet is as obnoxious to official religion as the last, for it declares that the stories, events, incidents and sayings, which in their totality constitute a holy scripture, are merely a mixture of imagined allegories and actual hap-

> penings, a literary concoction whereby mystical truths are cleverly conveyed through the medium of symbolic myth, legendary personification and true historic fact; that the twentieth century indeed could quite justifiably write its new Bibles, its new Qurans, its new Vedas afresh if it wished, for the divine afflatus may descend again at any hour.
> Mystics hold, fifthly, that their practices ultimately lead to the development of supernormal faculties and extraordinary mental powers or even strange physical ones, either as the gift of God's grace or as the consequence of their own efforts.
> Dr Paul Brunton, *The Hidden Teaching beyond Yoga*

Is there a route to knowledge — even "ultimate knowledge" — that lies outside the road of rational scientific enquiry and logical reasoning? Many people believe there is and call it mysticism. Most scientists have a deep mistrust of mystical thought, as it appears to lie at the opposite extreme to rational thought, the basis of the scientific method. Also, mysticism tends to be confused with the occult, the paranormal, and other fringe beliefs. In fact, many of the world's finest thinkers, including some notable scientists such as Einstein, Pauli, Schrödinger, Heisenberg, Eddington and Jeans, have also espoused mysticism in some form.

> The expression "mystical experience" is often used by religious people, or those who practice meditation. These experiences, which are undoubtedly real enough for the person who experiences them, are said to be hard to convey in words. Mystics frequently speak of an overwhelming sense of being at one with the Universe or with God, of glimpsing a holistic vision of reality, or of being in the presence of a powerful and loving influence. Most important, mystics claim that they can grasp ultimate reality in a single experience, in contrast to the long and tortuous deductive sequence . . . of the logical-scientific method of inquiry. Sometimes the mystical path seems to involve little more than an inner sense of peace — "a compassionate, joyful stillness that lies beyond the activity of busy minds" was the way a physicist colleague once described it to me. Einstein spoke of a "cosmic religious feeling" that inspired his reflections on the order and harmony of nature. Some scientists, most notably the physicists Brian Josephson and David Bohm, believe that regular mystical insights achieved by quiet meditative practices can be a useful guide in the formulation of scientific theories.
> Paul Davies, *The Mind of God*

Revelation

Many people, including physicists, have experienced moments of revelation, when a new and complete idea arises in the mind, in a flash of inspiration. When the Irish mathematician William Rowan Hamilton had such an experience while walking in Dublin, he was so excited that he scratched his new quaternian equations into the stone of Brougham Bridge. A complete and holistic understanding had arrived suddenly to make sense of problems he had been thinking about for a long time. He continued to work out the details of his discovery for the rest of his life.

Fred Hoyle described a revelatory experience he had while driving in northern England:

> As the miles slipped by I turned the quantum mechanical problem . . . over in my mind, in the hazy way I normally have in thinking mathematics in my head. Normally, I have to write things down on paper, and then fiddle with the equations and integrals as best I can. But somewhere on Bowes Moor my awareness of the mathematics clarified, not a little, not even a lot, but as if a huge brilliant light had suddenly been switched on. How long did it take to become totally convinced that the problem was solved? Less than five seconds. It only remained to make sure that before the clarity faded I had enough of the essential steps stored safely in my recallable memory. It is indicative of the measure of certainty I felt that in the ensuing days I didn't trouble to commit anything to paper.
> Fred Hoyle, quoted by Paul Davies in *The Mind of God*

Hoyle also recalled a conversation with Richard Feynman on the subject of revelation:

> Some years ago I had a graphic description from Dick Feynman of what a moment of inspiration feels like, and of it being followed by an enormous sense of euphoria, lasting for maybe two or three days. I asked how often had it happened, to which Feynman replied 'four', at which we both agreed that twelve days of euphoria was not a great reward for a lifetime's work.
> Fred Hoyle, quoted by Paul Davies in *The Mind of God*

The Supreme State

Although there is a consensus in the Perennial Philosophy, this does not mean that there is agreement about details. No one really seems to know what 'enlightenment' is, or how to tell if someone is enlightened or not. Osho wrote that Aurobindo, one of the great 20th century Indian saints, was not enlightened, even though Aurobindo's followers maintain that he was enlightened during his imprisonment by the British colonial rulers of India. There are also plenty of people who will say that Osho wasn't enlightened, even though his followers insist he was. . . There are many levels to be reached in mystic states and how do you label them reliably? Some Buddhists claim that Buddha reached higher states than anyone else, but it is highly unlikely that a devout Christian or Muslim would agree.

> The Buddha declined to make any statement in regard to the ultimate divine Reality. All he would talk about was Nirvana, which is the name of the experience that comes to the totally selfless and one-pointed. To this same experience others have given the name of union with Brahma, with Al Haqq, with the immanent and transcendent Godhead. Maintaining, in this matter, the attitude of a strict operationalist, the Buddha would speak only of the spiritual experience, not of the metaphysical entity presumed by the theologians of other religions, as also of later Buddhism, to be the object and (since in contemplation the knower, the known and the knowledge are all one) at the same time the subject and substance of that experience.
> Aldous Huxley, *The Perennial Philosophy*

The supreme state is not describable in words, it has to be experienced, but here are two attempts to provide what limited understanding words can convey. The first is from Plotinus the great Roman mystic and follower of Plato:

> There were not two; beholder was one with beheld; it was not a vision compassed but a unity apprehended. The man formed by this mingling with the Supreme must — if he only remember — carry its image impressed upon him: he is become the Unity, nothing within him or without inducing any diversity; no movement now, no passion, no outlooking desire, once this ascent is achieved; reasoning is in abeyance and all Intellection and even, to dare the word, the very self: caught away, filled with God, he has in perfect stillness attained isolation; all the being calmed, he turns neither to this side nor to that, not even inwards to himself; utterly resting he has become rest. He belongs no longer to the order of the beautiful; he has risen beyond beauty; he has overpassed even the choir of the virtues; he is like one who, having penetrated the inner sanctuary, leaves the temple images behind him — though these become once more first objects of regard when he leaves the holies; for There his converse was not with image, not with trace, but with the very Truth in the view of which all the rest is but of secondary concern.

> There, indeed, it was scarcely vision, unless of a mode unknown; it was going forth from the self, a simplifying, a renunciation, a reach towards contact and at the same time a repose, a meditation towards adjustment. This is the only seeing of what lies within the holies: to look otherwise is to fail.
> Plotinus (205 -270 AD), *The Enneads*

The second description of the supreme state is from the Spanish mystic and Roman Catholic saint John of the Cross.

I entered into unknowing
And there I remained unknowing
Transcending all knowledge.

1. I entered into unknowing
Yet when I saw myself there
Without knowing where I was
I understood great things;
I shall not say what I felt
For I remained in unknowing
Transcending all knowledge.

2. That perfect knowledge
Was of peace and holiness
Held at no remove
In profound solitude;
It was something so secret
That I was left stammering,
Transcending all knowledge.

3. I was so whelmed
So absorbed and withdrawn,
That my senses were left
Deprived of all their sensing,
And my spirit was given
An understanding while not understanding,
Transcending all knowledge.

4. He who truly arrives there
Cuts free from himself;
All that he knew before
Now seems worthless,
And his knowledge so soars
That he is left in unknowing
Transcending all knowledge.

5. The higher he ascends
The less he understands,
Because the cloud is dark
Which lit up the night;
Whoever knows this
Remains always in unknowing
Transcending all knowledge.

6. This knowledge in unknowing
Is so overwhelming
That wise men disputing
Can never overthrow it,
For their knowledge does not reach
To the understanding of not-understanding,
Transcending all knowledge.

7. And this supreme knowledge
Is so exalted
That no power of man or learning
Can grasp it;
He who masters himself
Will, with knowledge in unknow-
ing, Always be transcending.

8. And if you should want to hear:
This highest knowledge lies
In the loftiest sense
Of the essence of God;
This is a work of His mercy,
To leave one without understanding
Transcending all knowledge.

St. John of the Cross (1542 -1591)
Stanzas Concerning an Ecstasy Experienced in High Contemplation

The exponents of the Perennial Philosophy have shown that it is possible, although very difficult for most of us, to enter a completely different state of consciousness, free from all the normal worldly attachments. This state is somehow dual to our normal conscious state, and the highest aim is not to rest there, but to enjoy the best of both worlds, bringing the peace of eternity into the everyday world. This duality of consciousness, one part in time, and one part in eternity, is a clue to how the physics of the Universe works as we will see.

3
A Thinking Universe

Nothing Exists but Thoughts

> Nothing exists but thoughts! The universe is composed of impressions, ideas, pleasures, and pains.
> Humphry Davy, 1799, quoted in *A History of Science, Volume 4* by Henry Smith Williams

These were the first words that Humphry Davy managed to utter after tripping on nitrous oxide gas that he had inhaled as an experiment. A yogi follower of the Perennial Philosopher would say that he had witnessed the *Chitta*, or Mind-substance of the Universe.

> All existence, conscious or unconscious, is an EMANATION of one Being.
> The underlying "Being" has been called by many names by philosophers, the terms best adapted to it being "Spirit" or "The Absolute." The word "Absolute" is used in the sense "Unconditioned; Free from limitation; Complete in itself; Depending on nothing else; Actual; Real."
>
> The Three Great Manifestations of the Absolute are:
>
> 1) Matter, or Substance (Akasa)
> 2) Energy, or Force (Prana)
> 3) Mind-substance (Chitta)
>
> Yogis know that the three above mentioned manifestations are really not three, but are three phases of one manifestation, their teachings are that Matter is a grosser form of Energy or Force, gradually shading and melting into the latter; also that Force or Energy is a grosser form of Mind-substance, gradually shading and melting into this last mentioned manifestation. And the Mind-substance in its highest phases and operations almost reaches the plane of Spirit, from which it has emerged, in fact, it becomes so fine at the point of its emergence, that the human mind (even the mind of the most advanced souls) cannot point to the difference.
> Yogi Ramaracharaka, *Advanced Course in Yogi Philosophy*

As Yogi Ramacharaka was writing the above in 1905, a young man called Albert Einstein was deriving his famous equation connecting Matter and Energy. Towards the end of the 20th century, better understanding of the physics of information opened a new window into Chitta, the information processing Mind-substance that is inherent in everything.

Margolus-Levitin

The 1998 Margolus-Levitin theorem fixes the rate of computation in all energy or matter at 10^{51} logic operations per second per kilogram. This means that a single hydrogen atom, weighing just 10^{-27} kg, performs 10^{24} operations per second, more than all the computers built so far by humans put together. But there is yet more power in this natural logic. The processing rate found by Norman Margolus and Lev Levitin is derived from quantum mechanics, and counts operations on qubits. Whereas conventional computers operate with bits that can each take the value of 0 or 1, the qubits of a quantum computer can be both 0 and 1 at the same time, and all shades in between. It is only when a qubit is connected to the outer world by a measurement that it must choose a value of just 0 or 1. At present there is a world-wide race to build practical quantum computers, because theory predicts that they will be able to rapidly solve problems that are intractable for ordinary computers, and crack most of the encryption that is in use today. A quantum computer follows not one path at a time, but all paths simultaneously. The Universe is not just a giant computer, it is a giant quantum computer.

Margolus-Levitin tallies up the simple NOT operations of individual bits, represented within all energy and matter, flipping endlessly between 0 and 1. Interestingly, whereas the NOT operation of a conventional computer is trivial, a perfect quantum NOT operation on a qubit is known to be impossible. This is a consequence of the no-cloning theorem which says that a qubit cannot be copied. The Universe, it seems, has no choice but to evolve continually, with each flip of one of its qubits.

Grain of Sand or Supercomputer?

> To see a World in a Grain of Sand
> And a Heaven in a Wild Flower
> Hold Infinity in the palm of your hand
> And Eternity in an hour.
> William Blake, *Auguries of Innocence*

While Blake's words are well known, how many people can share his vision?, and it certainly does not sound scientific. The efforts of physicists who study the physics of information processing and the ultimate limits of computation, have made it clear that, despite the great progress already made, our latest technologies are just scratching the surface of what is possible, and of what nature is already using. We can illustrate this by looking at the computing capacity of William Blake's grain of sand.

In 2018, the title for fastest computer in the world was held by IBM's Summit, which can perform 10^{17} calculations per second. It is estimated that the total computing power of all the world's computers put together is currently around 10^{19} operations per second and doubling every 3 years. In comparison, a grain of sand weighing 1 milligram clocks about 10^{45} logical operations per second, and outperforms all the world's processors combined by a factor of 10^{26}. If you are a geek who gets excited about the performance of the latest computers, pondering the extreme hidden power of this natural information processing may just give you a hint of Blake's vision. We, and the reality we live in, are made of supercomputing material with capabilities very far beyond our comprehension. If you object that you cannot see anything going on when you look at a grain of sand, remember that the same applies to any silicon chip, and that the chip is just made from — sand.

The big question is: what is all this logical processing doing? 20th century physics buried such considerations by dismissing this intricate intelligence as randomness. In his book *Programming the Universe*, Seth Lloyd writes that the Universe computes itself, and while this may be true, it begs the question of what the extraordinary computing power built into apparently inert matter is doing, there surely must be much more going on — does a grain of sand really need all that internal computing power just to behave like a grain of sand? As a schoolboy, one of the principal objections I had to believing in a God that knew everything and saw everything, was that physics appeared to make such things impossible, but subsequent advances in technology, and the physics of information have changed the picture dramatically. We now know that there is almost limitless computing power everywhere, and in everything. We have also been surprised many times by the world changing powers of our still primitive computing devices, and there is a lot more to come from our own technology.

You may argue that not all types of logical operations are equivalent, the floating point processing of our supercomputers is more powerful than the simple bit flipping NOT operations counted by Margolus-Levitin, but give or take one or two factors of ten, the above is a fair comparison. You also have to take into account that nature's own processing uses reversible computing which wastes no energy, and quantum qubits which can represent all possibilities simultaneously rather than just the binary pair of possibilities of a normal bit. Both these enhanced forms of computing are still beyond our current technology, but the race to build quantum computers is in full swing as they have the potential to quickly solve problems that would take aeons on our current computers.

Seth Lloyd has calculated the following statistics for the Universe as computer:

Current speed:	10^{105}	logical operations per second
Memory:	10^{92}	bits
Total ops performed so far:	10^{122}	logical operations

The value of 10^{92} bits for the memory is found from thermodynamic arguments which attribute one bit of memory to each fundamental particle in the Universe. The holographic principle, which we look at in Chapter 12, suggests that the Universe represents all the 3D information it contains in dual form as 10^{122} bits on its 2D boundary surface. Lloyd suggests that the discrepancy between the two estimates of the Universe's memory arises because by far the greater part of the Universe's memory is in gravitational degrees of freedom that the holographic principle counts, but we so far do not understand.

Laplace's Demon

> We may regard the present state of the Universe as the effect of its past and the cause of its future. An intellect which, at a certain moment, would know all forces that set nature in motion, and all the positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the Universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future, just like the past, would be present before its eyes.
> Pierre Simon Laplace, *A Philosophical Essay on Probabilities*

Seth Lloyd sees a couple of problems with Laplace's great intellect or 'demon'. The first is that the demon would have to use as much space, time, and energy as the Universe itself, to have sufficient computational resources. The second problem is:

> . . . the laws of quantum mechanics are not deterministic in the sense required by Laplace. In quantum mechanics, what happens in the future is predictable only in a probabilistic way. In fact, the motions of heavenly bodies are intrins-

> ically chaotic and thus are constantly pumping information up from microscopic to macroscopic scales.
>
> . . . the result of this celestial chaos is that even Laplace's heavenly bodies move in a probabilistic fashion that cannot be predicted, even by a demon.
> Seth Lloyd, *Programming the Universe*

The God of Randomness

Lloyd follows the usual 'dice playing' interpretation of quantum mechanics, but makes it clear that quantum processes are responsible for the creative process of introducing 'new' information into the Universe, even though it is the physicist's God of randomness that is the creative source.

> The laws of quantum mechanics are responsible for the emergence of detail and structure in the universe.
>
> The theory of quantum mechanics gives rise to large-scale structure because of its intrinsically probabilistic nature. Counterintuitive as it may seem, quantum mechanics produces detail and structure because it is inherently uncertain.
>
> . . . The miniscule quantum fluctuations of energy density at the time of the Big Bang are the butterfly effects that would come to yield the large-scale structure of the universe.
>
> . . . Every galaxy, star and planet owes its mass and position to quantum accidents of the early universe. But there's more: these accidents are also the source of the universe's minute details: Chance is a crucial element of the language of nature. Every roll of the quantum dice injects a few more bits of detail into the world. As these details accumulate, they form the seeds for all the variety of the universe. Every tree, branch, leaf, cell, and strand of DNA owes its particular form to some past toss of the quantum dice. Without the laws of quantum mechanics, the universe would still be featureless and bare.
> Seth Lloyd, *Programming the Universe*

All structure at macroscopic scales derives from information 'pumped up' from the quantum scale of atoms, molecules and elementary particles. At the quantum scale there is superposition of information, all possible outcomes of events are logically mapped out, but what actually selects the outcome that emerges is a complete mystery. The majority of physicists are atheists who see it as a dice game because it appears so perfectly random, but some physicists, like Bohm, believe that creative guidance comes from a deeper source, in hidden variables that the Copenhagen interpretation rejected.

The Bhagavad Gita also distinguishes between the causal processes of Nature, and a deeper, creative source, the Lord of this world.

> Neither the sense of acting, nor actions, nor the connection of cause and effect comes from the Lord of this world. These three arise from Nature.
> Bhagavad Gita, 5, 14

> I am the source of all; from Me everything flows. Therefore the wise worship Me with unchanging devotion.
> Bhagavad Gita, 10, 8

Entropy

The term ‘entropy’ was introduced by Rudolf Clausius in 1850 to account for the heat energy lost, or wasted, by a steam engine. Physicists of that time were trying to understand the theory of steam engine operation in order to improve their efficiency. This was the origin of the science of *thermodynamics*. Entropy is defined as the ratio of energy to temperature, and is a measure of how spread out and useless the energy is. Hot steam can expand and push a piston, cool steam that has expanded is useless. A certain amount of energy at a high temperature has a low entropy, and is useful for doing work, but the same quantity of energy at a low temperature has a high entropy, and is useless for doing work. Heat energy likes to spread out, flowing naturally from hot bodies to cooler bodies, obeying the second law of thermodynamics which says that entropy tends to increase.

In 1875, Ludwig Boltzmann discovered the formula $S = k\log W$ that connects entropy with the motions of gas molecules. S is the entropy, W is the number of possible positions and velocities available to the molecules, and k is Boltzmann's constant. Boltzmann’s equation says that entropy, is the amount of ‘hidden information’ representing the positions and velocities of gas molecules. The formula $S = k\log W$ is inscribed on Boltzmann’s tomb.

In 1948, Claude Shannon published the landmark paper, *A Mathematical Theory of Communication*, showing that the amount of information in a message obeys the very same equation as Boltzmann’s thermodynamic entropy. Accordingly, he used the term entropy to describe the information content of a message.

> Legend has it that Shannon adopted this term [entropy] on the advice of the mathematician John Von Neumann, who declared that it would give him ". . .

> a great edge in debates because nobody really knows what entropy is anyway."
>
> Richard Feynman, *Lectures on Computation*

Entropy is often described as a measure of disorder, and it is true that a randomly disordered message contains more information than a simple string of repetitions of one character. But disorder is easy to confuse with meaninglessness, and this is dangerous; just because some information is meaningless to you does not mean it lacks any meaning in the greater scheme of things, we hide our messages by encrypting them to make them appear random. Shannon certainly was not using entropy to describe meaningless information — why would someone want to send a meaningless message? The Billiard Ball model of computing that we look at in Chapter 9, is very reminiscent of the endless jostling collisions of molecules that are driven by their heat energy. Physicists describe the information processed by jostling atoms as entropy, or meaningless disorder, but is this justifiable? We look at the subject of randomness and its confusions in Chapter 6.

The Laws of Thought

The invention of the computer has been attributed to many different people, and particularly Charles Babbage for his 1837 design of the *analytical engine*, a programmable mechanical computer that he never managed to build, owing to funding problems and limitations in the precision of 19th century machining technology. On the theoretical side, it was George Boole who gave the first method for processing logical expressions using the symbols of algebra rather than a language such as English. Boole's 1854 book *An Investigation of the Laws of Thought* was later described by Bertrand Russell as "the birth of pure mathematics".

Computer science students learn Boolean algebra, which uses the fundamental logical operations of AND, OR and NOT as its fundamental building blocks. Any 'universal' computer can be made by just connecting together a number of these individual logic 'gates', and we will look at this essential simplicity of computers in Chapter 9. The method Boole presented in the *Laws of Thought* is quite different from this. Modern Boolean logic has lost the holistic quality of Boole's original method which relies on taking background information into account, as well as foreground information, the NOT-something plays its shadowy part along with the something. Boole also emphasised the need to refer all logic to the background of Universe and Eternity:

> “We cannot perfectly express the laws of thought, or establish in the most general sense the methods of which they form the basis, without at least the implication of elements which ordinary language expresses by the terms “Universe” and “Eternity.” As in the pure abstractions of Geometry, so in the domain of Logic it is seen, that the empire of Truth is, in a certain sense, larger than that of Imagination.”
> George Boole, *The Laws of Thought*

The last statement is a foresight of the famous incompleteness theorems of Kurt Gödel that shook the world of mathematics in 1931. Mathematicians had been striving to create a finite set of axioms from which all of mathematics would follow, without any contradictions and loose ends, but Gödel showed that whatever system of logic you use, there will always remain truths that you cannot prove because they are beyond your system.

Boole uses an algebraic symbol such as x, to represent a class of things such as men: x = “men”. There then has to be the class of NOT-men given by 1-x. Universe is represented by 1, so 1-x means the rest of the Universe when you take x away from it. Fig. 3.1 shows the same idea in diagrammatic form.

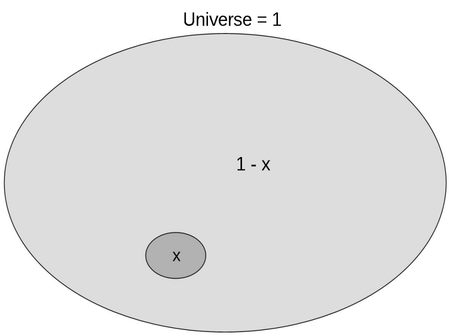


Figure 3.1 Boole's NOT

The dimension of scale provides a natural separation between a something and a NOT-something. Any object, whether it is an atom, an apple or a planet, has some sort of 2 dimensional boundary surface that separates inside from outside in the scale dimension, which we may take as running from the centre of the object out to infinity. If you call what is within the bounding surface of the object x, then what is outside the object is 1-x where 1 is the whole of the rest of the Universe. We will see later that this logical NOT process of inside-outside reversal is going on in all the physical entities of the Universe, but when the separation is in the scale dimension, the process seems to be using a form of inversion of x with 1/x rather than a simple x and 1-x.

Boole uses two symbols written together as a product to mean that both are true at the same time, so if x = “white things” and y = “sheep”, the product xy means “white sheep”. The law of thought is just the equation $x^2 = x$, which can be re-arranged to the form x(1-x) = 0. In this form the equation is saying that the class of things that are at once men and not-men does not exist. Boole observes that this is:

> “...that principle of contradiction which Aristotle has described as the fundamental axiom of all philosophy. It is impossible that the same quality should both belong and not belong to the same thing... This is the most certain of all principles... Wherefore they who demonstrate refer to this as an ultimate opinion. For it is by nature the source of all the other axioms.”
> George Boole, *The Laws of Thought*

The only numbers that are equal to their own squares and satisfy the law of thought are 0 and 1, which Boole called Nothing and Universe. In Boole’s peculiar algebra the x’s and y’s can only take on the values 0 and 1, they are binary variables. In a sense Boole had discovered the bit, but it took another century to name it, and put it to work in computers.

Boole’s system of logic has been found slightly unsound under modern analysis, and I have included this brief look at it only because of his emphasis of the importance of the NOT-TRUE along with the TRUE, and the quality of apparently divine inspiration that drove his work along with that of so many great scientists.

> “To infer the existence of an intelligent cause from the teeming evidences of surrounding design, to rise to the conception of a moral Governor of the world, from the study of the constitution and the moral provisions of our own nature; — these, though but the feeble steps of an understanding limited in its faculties and its materials of knowledge, are of more avail than the ambitious attempt to arrive at a certainty unattainable on the ground of natural religion. And as these were the most ancient, so are they still the most solid foundations, Revelation being set apart, of the belief that the course of this world is not abandoned to chance and inexorable fate.”
> George Boole, *The Laws of Thought*

A worked example of Boole’s method from *The Laws of Thought* is included in Appendix 1.

The Cosmic Computer

God is traditionally credited with powers that are far greater than what physics appears to allow, and this is often taken as proof of God's non-existence. I felt my own spirituality constrained by the laws of physics for many years, and Richard Dawkins has used this argument in favour of atheism, as we saw in Chapter 2. Here again is part of what Dawkins wrote:

> . . . a God who is capable of sending intelligible signals to millions of people simultaneously, and of receiving messages from all of them simultaneously, cannot be, whatever else he might be, simple. Such bandwidth! God may not have a brain made of neurons, or a CPU made of silicon, but if he has the powers attributed to him he must have something far more elaborately and non-randomly constructed than the largest brain or the largest computer we know.
>
> Richard Dawkins, *The God Delusion*

Such bandwidth indeed. And God has even more to do than just maintaining communications with Earthbound humans, he-she-it has a whole Universe to service, with billions of galaxies, each containing billions of stars, and untold numbers of creatures. Fortunately, Nature's I.T. is just as extreme as the powers attributed to God. It uses quantum computation that can consider all possibilities at the same time, and operates in every mass-carrying particle at a speed set by Margolus-Levitin, fast enough for a single electron to outclock all the world's silicon chips put together. Nature also possesses, in the quantum phenomenon of entanglement, a technology that allows instant synchronisation of information across space. And then there is gravity, which physicists have been trying to marry with quantum mechanics for nearly 100 years without success. Roger Penrose and some other physicists believe that gravity is the final decision maker in the outcome of quantum measurements, not the God of randomness supported by mainstream physics. We look further at this in Chapter 14.

The depiction of God as cosmic computer in this book is not meant to demean God, or imply that this is all there is to the divine One, it is merely a recognition of what is accessible to our understanding in terms of today's physics, and an attempt to remove much of the unnecessary and illogical thinking that corrupts the world's religions. The separation of God into different levels of function is also common in theology, and is part of the Perennial Philosophy. This is particularly clear in Hinduism-Yoga, where the One is subdivided into the trinity of Gods: Krishna the creator/destroyer, Vishnu the preserver, and Brahma who does nothing much other that occupy the central reference point of creation. The *Bhagavad Gita*, the *Upanishads*, and *Patanjali's Yoga Sutras*, all over 2000 years old, are closely related expressions of the Perennial Philosophy, which offer the best source for understanding the physical and spiritual engineering of the Universe.

The Bhagavad Gita is the most popular spiritual text in India and was Gandhi's favourite source of reference throughout his life. In the Gita, Krishna appears in human form to explain to prince Arjuna the relationship of God and man, the workings of God and the Universe, and the levels of God's own nature. The following quotations are selected for their relevance to the idea that Nature works as a purely logical and mechanical process, on a different level from the pure Self or God at the centre of the Universe.

> Earth, water, fire, air, aether, mind, intellect and personality; this is the eightfold division of My Manifested Nature.
> This is My inferior Nature; but distinct from this, O Valiant One! know thou that my Superior Nature is the very Life which sustains the universe.
> It is the womb of all being; for I am He by Whom the worlds were created and shall be dissolved.
> O Arjuna! There is nothing higher than Me; all is strung upon Me as rows of pearls upon a thread.
>
> . . . The Lord of this universe has not ordained activity, or any incentive thereto, or any relation between an act and its consequences. All this is the work of Nature.
> It is only a very small part of My Eternal Self, which is the life of this universe, drawing round itself the six senses, the mind the last, which have their source in Nature.
>
> . . . The whole world is pervaded by Me, yet My form is not seen. All living things have their being in Me, yet I am not limited by them.
> Nevertheless, they do not consciously abide in Me. Such is My Divine Sovereignty that though I, the Supreme Self, am the cause and upholder of all, yet I remain outside.
> As the mighty wind, though moving everywhere, has no resting place but space, so have all these beings no home but Me.
> All beings, O Arjuna! Return at the close of every cosmic cycle into the realm of Nature, which is a part of Me, and at the beginning of the next I send them forth again.

With the help of Nature, again and again I pour forth the whole multitude of beings, whether they will or no, for they are ruled by My Will.
But these arts of Mine do not bind Me. I remain outside and unattached.
Under My guidance, Nature produces all things movable and immovable. Thus it is, O Arjuna! that this universe revolves.

. . . Know thou further that Nature and God have no beginning; and that differences of character and quality have their origin in Nature only.

. . . Nature is the Law which generates cause and effect; God is the source of the enjoyment of all pleasure and pain.
God dwelling in the heart of Nature experiences the Qualities which Nature brings forth; and His affinity towards the Qualities is the reason for His living in a good or evil body.
Thus in the body of man dwells the Supreme God; He who sees and permits, upholds and enjoys; the Highest God and the Highest Self.
Bhagavad Gita

The mystic Roman philosopher Boethius, 480-524 AD, wrote *The Consolation of Philosophy*, which became one of the most popular and influential works of the Middle Ages. What follows is his description of a two level model of God's action in providing for the inhabitants of creation. Providence is the higher level, timeless, and closer to the divine; Fate, or karma, is the lower level, comprising the detailed, material unfolding of events in time.

The manifest world and whatever is moved in any sort take their causes, order and forms from the stability of the divine Mind. This hath determined manifold ways for doing things; which ways being considered in the purity of God's understanding are named Providence; but being referred to those things which he moveth and dispotheth are called Fate. . . . Providence is the very divine Reason itself, which dispotheth all things. But Fate is a disposition inherent in changeable things, by which Providence connecteth all things in their due order. For Providence equally embraceth all things together, though diverse, though infinite; but Fate puts into motion all things, distributed by places, forms and times; so that the unfolding of the temporal order, being united in the foresight of the divine Mind is Providence, and the same uniting, being digested and unfolded in time, is called Fate. . . . As a workman conceiving the form of anything in his mind, taketh his work in hand and executeth by order of time that which he had simply and in a moment foreseen, so God by his Providence dispotheth whatever is to be done with simplicity

and stability, and by Fate effecteth by manifold ways and in the order of time those very things which he dispotheth. . . . All that is under Fate is also subject to Providence. But some things which are under Providence are above the course of Fate. For they are those things which, being stably fixed in virtue of their nearness to the first divinity, exceed the order of Fate's mobility.
Boethius, quoted in *The Perennial Philosophy*, Aldous Huxley

Dependent Origination

Gautama Buddha also provided in his law of *Dependent Origination*, *Pratītyasamutpāda*, a description of Nature as a purely mechanical, twelve step process, operating below the divine level. Buddha said that this was his deepest teaching, and it came to Buddha on the night of his enlightenment. Buddhist monks still chant its steps backwards and forwards 2,500 years later. It is a difficult teaching to understand, as Buddha himself acknowledged, and there are many ways of describing the twelve steps; here are two:

Paticcasamuppada —	**Dependent Origination**
Ignorance	Nibbana
Mental Concocting	Knowledge of the Deliverance
Consciousness	Deliverance
Mind/Body	Fading Away
Sense Bases	Disgust
Contact	Knowledge of How Things Are
Feeling	Concentration
Craving	Happiness
Attachment	Tranquility
Becoming	Rapture
Birth	Joy
Old Age, Suffering — Death	Faith

The sequence — contact, feeling, craving, attachment — is perhaps easiest to understand, as it describes the onset of addiction. Buddha said that it is possible, with greater self-awareness, to recognise the progression of these steps in our own consciousness, and stop the cycle. The point here is simply to show another example in the Perennial Philosophy of a strictly logical process that might be running on a computer.

Reverse Engineering the Cosmic Computer

To help with the task of later chapters of reverse-engineering the Cosmic Computer, we may try to identify some of the functions that it has to perform in the everyday running of the Universe, servicing all its constituent creatures while allowing them free will. If form follows function, this should help us understand the Cosmic Computer's architecture. According to the Perennial Philosophy, the most important functions include:

- Love
- Instantaneous cosmic communications
- Providence
- Assisted learning
- Cosmic justice enforcement through karma
- Creativity
- Entertainment

4
Number

One is the Number

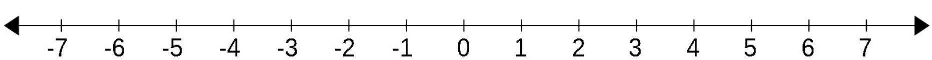
> . . . thou understandest that 1 is no number, but it is a generatrix, beginning, and foundation for all other numbers.
> Jacob Köbel, 1537

We are so familiar with our decimal numbers, and they are so useful, that we take them for granted and do not question how they are generated, but the truth is that they all must start from 1. As some mathematicians, like Köbel, have realised, it is important to recognise the special nature of 1, failure to do this has led us astray and this can account for some of the problems in mathematics and physics.

> The Greeks did not consider 1, or unity, to be a number at all. It was the monad, the indivisible unit from which all other numbers arose. According to Euclid a number is an aggregate composed of units. Not unreasonably, they did not consider 1 to be an aggregate of itself.
> David Wells, *The Penguin Dictionary of Curious and Interesting Numbers*

> God made the integers, all the rest is the work of man.
> Leopold Kronecker, 1823-1891

The simplest number system in mathematics is the set of natural numbers: 1, 2, 3, . . . , what you can count, and some mathematicians still argue that these are the only "true" numbers. The Indian mathematician Brahmagupta is credited with the first use of the negative numbers which only have meaning in that they can be subtracted from natural numbers. The zero was also born in India and conveyed to Europe by Arab mathematicians in the 11th century A.D. This completed what we now call the integers which are often represented on a number line:



Real Numbers

Rational numbers like the fraction 3/4 were known to the ancient Greeks, but it was not until the 16^{th} century that the modern decimal system of real numbers came into use. To fill in the gaps between the integers in the number line, you need not just multiples of 1, but also subdivisions of 1. The real numbers are so ubiquitous and useful that we take them for granted, despite the fact that a number like 4.3 is a strange, two compartment affair. To the left of the decimal point we have the number of multiples of the unit 1, and to the right we have the number of sub-divided pieces of the unit 1, where for example .3 means 3 of the pieces of a new smaller unit you obtain by dividing the unit 1 by 10.

This may seem picky, but it is crucially important in what follows to grasp that there are fundamental differences between multiples of a unit and divisions of a unit. Multiplying 1 by a million takes you 999,999 units further to the right along the number line. Dividing 1 by a million will not even quite manage to reach 0, the next unit to the left of 1. The number line is fine for addition and subtraction, but can be very deceptive for multiplication and division.

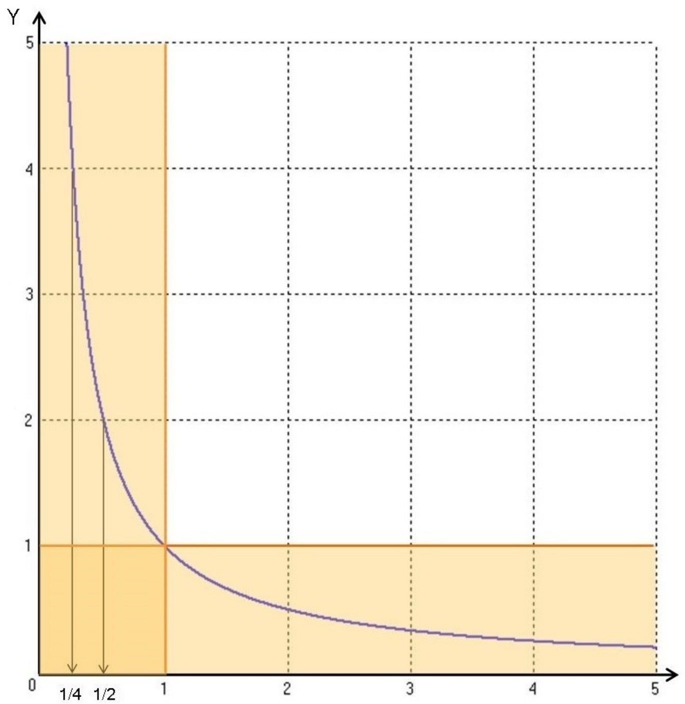


Fig. 4.1 Graph of y=1/x

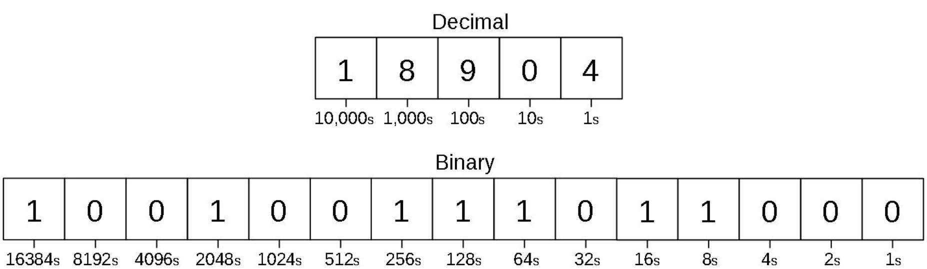
In terms of the amount of information, it takes as many digits to represent 1 millionth, 0.000001, as it does to represent one million, 1000000. The number you are representing may get smaller, but the size of the representation must increase. When you divide the unit 1 to represent say a half or a quarter, you are effectively venturing into an orthogonal, sideways dimension, as illustrated by the y = 1/x as opposed to the x in Fig. 4.1.

The curve is the hyperbola given by y = 1/x. Because y = 1/x, xy = 1, showing that the product of the x measure and the y measure is always the unit 1. Notice how, for all values of x greater than 1, the y value of the hyperbola stays in the lightly shaded area less than 1, and for all values of y greater than 1, the x value stays less than 1. The unit again is invariant; the product xy, x times y, is always 1.

Real numbers are stored in computers as floating point numbers, which have two parts, a *mantissa*, and an *exponent*. The mantissa contains the non-zero part of the number, and the exponent keeps the count of how many places to shift the decimal point to the left or right. So, for example, when we write Planck's constant as 6.63 x 10^{-34}, the mantissa is 6.63, and the exponent is 10^{-34}, and this notation is far simpler for humans and machines than writing:

0.000000000000000000000000000000000663.

Binary Numbers



The number 18904 in decimal and binary

Fig 4.2 Decimal and binary numbers

While humans use decimal numbers, computers prefer to think in binary. It may require more symbols — the number 18904 above requiring 15 bits rather than 5 decimal digits — but binary only needs the two state symbols 0 and 1, rather than the ten symbols 0,1,2,3,4,5,6,7,8,9, and this makes binary more efficient, and greatly simplifies computer construction. Just as each digit in a decimal number has ten times the value of the one to its right, each bit added to the left of a binary number has double the value of the one to its right. Because each octave on a piano represents a doubling of frequency, an octave is also a sort of bit. The 61 powers of 10, 10^{61}, radius of the Universe in the scale dimension can also be seen as 202 powers of 2, 2^{202}. As far as scale is concerned, we live in a 202 bit Universe. Current computers generally use 64 bit registers to store numbers and data.

Imaginary Numbers

Despite the usefulness of real numbers, mathematicians still found that there were some equations whose solutions could not be written down. It turned out that they only needed one more quantity to complete their numerical toolkit: the square root of -1, $\sqrt{-1}$, which is usually written as i. Whereas real numbers are based on the unit 1, imaginary numbers use i as their base unit, so the imaginary number 5i is just 5 times the imaginary unit i. The label 'imaginary' does not mean that imaginary numbers are unreal, they are vital for describing the mathematics of reality, but they live in a separate dimension to real numbers. It can be argued that the imaginary unit i is even more fundamental than the unit 1, as it is easier to obtain 1 from i, than it is to go the other way. You can get 1, -i, or -1, simply by multiplying i by itself:

$$i^1 = i$$

$$i^2 = i \times i = -1$$

$$i^3 = i \times i \times i = -i$$

$$i^4 = i \times i \times i \times i = 1$$

So we see that i is a 4th root of 1, and so are -i, 1 itself, and -1:

$$i^4 = (-1)^4 = (-i)^4 = 1^4 = 1$$

The fact that our plain and simple unit 1 has these four 'secret' inner compartments plays a fundamental role in the workings of the Universe, and makes the quotations at the start of this chapter seem a little short sighted. We will see later that the Dirac equation requires an electron's wave function to have four component parts that describe aspects of the electron with positive and negative mass, and up or down spin. Dirac's equation is one of the greatest triumphs of 20th century physics. Unlike the Schrödinger equation, it is relativistically correct, it incorporates spin, and allowed Dirac to predict the existence of anti-matter, it is also the main foundation for quantum electrodynamics, and governs the electron's hidden zitter dance that we peek at in Chapter 13.

When the Margolus-Levitin theorem counts logical operations per second, it is counting cycles around the four revolving compartments of the unit 1. In our 'real' outer world we don't see these inner 'imaginary' dimensions, or phases, but they play a vital role in the mathematics of quantum mechanics.

Complex Numbers

Numbers measured in imaginary units i are especially useful when paired up with real numbers to form 'two compartment' *complex numbers*, which each have a real part, and an imaginary part, e.g. 4 + 3i. Complex numbers can be plotted on a 2D plane, called the complex plane, with the real numbers represented horizontally and the imaginaries vertically. This also makes clear how the imaginary part lives in a perpendicular, or 'orthogonal' dimension. Figure 4.3 shows how the operation 'multiply by i' is equivalent to 'rotate through a right angle', and clarifies the meaning of $i^2 = -1$. Four applications of 'multiply by i' gives a complete 360° rotation around the unit circle.

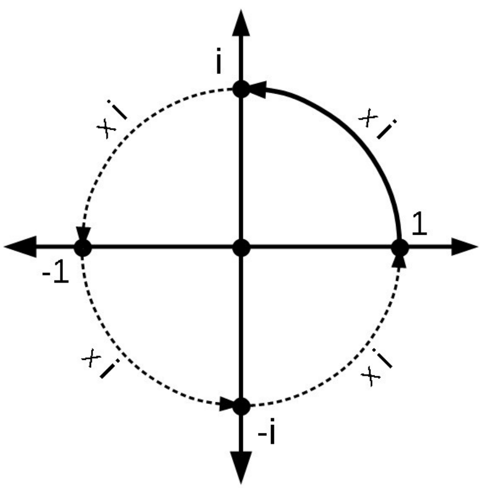


Fig. 4.3 The unit circle

Figure 4.4 shows the complex number 4 + 3i represented as a point in the complex plane. It also shows how the Pythagorean relation $3^2 + 4^2 = 5^2$ is deeply connected with the trigonometric functions Sine, Cosine and Tangent. The complex number 4 + 3i can equally well be described as the distance 5 units from the origin, and an angle 37 degrees. This 'polar form' of a complex number is important in quantum mechanics, where the angle measures its phase.

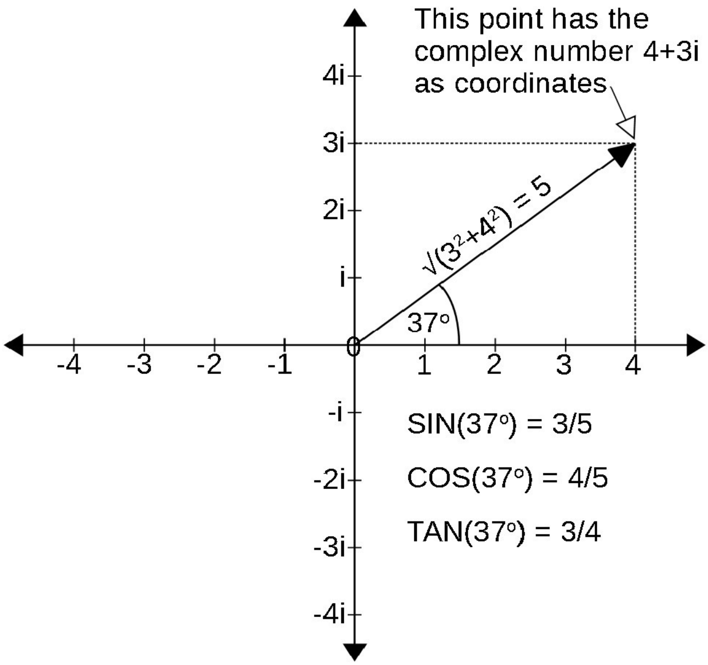


Fig. 4.4 Pythagoras & trigonometry in the complex plane

Squaring a Complex Number

One of the strange rules for a complex number is how you find its square. To square the complex number a + bi, you multiply it, not by itself, but by its *conjugate*, a – bi, the same complex number with its imaginary part flipped in sign between ±. Looking at the diagram above, you can see that taking the conjugate of a complex number is equivalent to taking its reflection in the real horizontal axis.

$$(a + bi) \times (a - bi) = a^2 - abi + bia - b^2i^2 = a^2 + b^2$$

The -abi and +bia add up to 0 and disappear, and i^2 is -1 by definition, so that $-b^2i^2$ is $+b^2$. The 'square' of a + ib is thus $a^2 + b^2$, which is completely real, the imaginary part having disappeared. This is the vital mathematical process in quantum mechanics that permits the real probability of measurement outcomes to be predicted by squaring the complex wave function. We may well wonder why quantum mechanics requires this peculiar product of a wave function with its mirror image in the real axis.

A probability must always be real, and positive, and in the range from 0, meaning no chance of happening, to 1, meaning certain to happen. No one knows what a negative or imaginary probability might mean, but before the squaring process, these complex 'probabilities' are called *amplitudes*, and it is these complex amplitudes that can interfere with each other and behave like waves. Quantum mechanical probabilities calculated in this way give perfect predictions for the odds of outcomes in the real outer world. Although the phase information in the imaginary channel is considered meaningless in the real outer world, we are starting to build quantum computers that depend on it to work their magic. We are, on the one hand saying that the quantum phase information is garbage, while on the other hand trying to harness it to supercharge our computers!

The Complex Exponential

The number 2.71828…is a fundamental constant in mathematics called e, or Euler's number. It has many special properties, one being that it is the only number which is equal to its own rate of change when its exponent varies. If the exponent is negative, the rate of change also becomes negative. This can be written

$$\frac{d}{dx}e^{x}=e^{x} \qquad \text{and} \qquad \frac{d}{dx}e^{-x}=-e^{-x}$$

An ordinary exponential like e^x, where e is raised to the power x, grows very rapidly as the exponent x increases. This behaviour is what we think of when we talk of exponential growth. For negative x, you get exponential shrinkage. If you change the exponent x to a complex number x + iθ, you get the complex exponential $e^{x+i\theta}$, which can also be written as the product of e^x and $e^{i\theta}$, $e^x e^{i\theta} = e^{x+i\theta}$. The e^x part is just a real exponential that scales the product up or down, but the $e^{i\theta}$ part behaves completely differently, being only concerned with rotation. As the angle θ changes, $e^{i\theta}$ can only cycle around the unit circle, passing through the four roots of 1 that we saw above: 1, i, -1, -i.

The de Moivre formula below shows how the $e^{i\theta}$ part is equivalent to a real cosine component and an imaginary sine component:

$$e^{i\theta} = \cos\theta + i\sin\theta$$

. . . a formula which some mathematicians consider one of the most remarkable and mysterious in all of mathematics, linking two branches of mathematics – trigonometry and the computation of compound interest – that existed entirely independently of each other for more than two thousand years.

. . . when we think of θ as the angle theta, the number $e^{i\theta}$ turns eternally around a circle of radius 1.

. . . Like π, e is irrational, in fact transcendental; like π, it tends to appear in the most unexpected places, like the de Moivre formula above or in the computation of interest. (If you put a dollar in the bank at 100 percent interest per year, compounded not every day or even every second, but continuously, at the end of a year it would be worth \$e = \$2.7182818539...)
Barbara Burke Hubbard, *The World According to Wavelets*

Setting the angle θ in the de Moivre formula above to π (the equivalent angular measure in radians to 180 degrees) leads to what is for many mathematicians the most wonderful equation in mathematics, the Euler formula which relates the five fundamental numbers 0, 1, i , π, and e in a simple, but quite mystical expression:

$$e^{i\pi} + 1 = 0$$

The fundamental number e = 2.71828..., is the base of natural logarithms. π = 3.1415..., is the ratio of the circumference of a circle to its diameter, and i is the square root of -1. Why should the unit imaginary number, angles and logarithms be related in such a simple formula? Nobody knows!

A.C. Power Example – The Importance of Phase

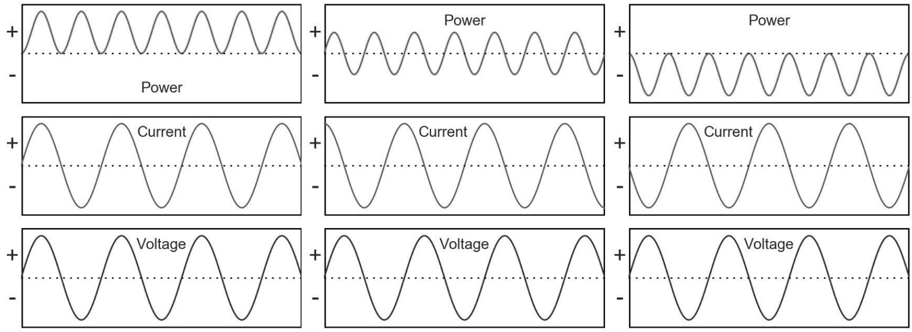


Fig 4.5 Current and voltage phase relations can give +ve or -ve AC power

Complex numbers are used routinely throughout science and engineering. They are very useful in electrical engineering where the two compartments of complex numbers allow them to keep track of the phases of alternating currents and voltages as well as their magnitudes. The power delivered by an A.C. electrical grid depends on the product of voltage and current in each time step, and both voltage and current oscillate up and down at the grid frequency of 50 or 60 Hz. If the voltage and current are in phase with each other, you get real power, measured in Watts (Volt.Amps). If the voltage and current are out of phase, you get imaginary, or reactive power, measured in Vars (Volt.Amps.Reactive). The load cannot consume energy in this case, it can only store it in electric or magnetic fields, and then return it to the power grid a fraction of a cycle later. Power companies have to minimize the proportion of reactive power, because it can be very destructive to equipment. The currents and voltages are real and cause losses in the grid, although no real power gets delivered to the customer. Industrial consumers of power are penalised if their equipment drags the voltage and current out of phase with each other, and often need extra equipment to correct this. The diagrams in Fig. 4.5 illustrate how the relative phases of voltage and current affect power.

Each power curve at the top is the product of the two curves below — the power is equal to voltage times current at every instant of time across the page. The diagram on the left shows that when the voltage and current are in phase, the power is always positive and real, flowing from generator to load. (The power fluctuates between zero and its maximum value here because we are only considering single phase AC power, rather than the 3-phase system that is used in electrical grids to deliver a constant level of power). The middle diagram shows that when the voltage and current are ¼ cycle out of phase, corresponding to the difference between a sine and a cosine wave, the power is reactive, fluctuating between positive and negative, flowing back and forth between generator and load, and averaging zero. The diagram on the right shows the voltage and current in anti-phase, ½ cycle out of phase with each other, and the power is always negative, it flows from the load back to the generator.

The Matrix

We have already seen that in the development of mathematics, it was found necessary to introduce numbers consisting of more than one 'compartment' in the form of complex numbers. This can be taken further so that you assemble a whole array of numbers in the form of a matrix. Matrices are of central importance in mathematics, and are particularly useful for representing linear transformations.

When Heisenberg was developing the first version of quantum mechanics in 1925, he found it necessary to manipulate arrays of numbers representing the energy levels of lines in the spectrum of light emitted by heated atoms. His method seemed to work, but he was troubled by the fact that the order in which he multiplied his arrays mattered: array X times array Y did not give the same answer as Y times X, and this was quite contrary to the normal rules of arithmetic. When he showed his work to Max Born, he too was equally intrigued and puzzled, and spent several days obsessively thinking about this strange multiplication law, it seemed vaguely familiar… Finally, he remembered a lecture that he had attended as a student, and realised that Heisenberg had simply reinvented matrix multiplication. When told about it, Heisenberg admitted that he had never heard of matrices.

Heisenberg's version of quantum mechanics came to be called *matrix mechanics*. Matrix multiplication is non-commutative, the order of multiplication matters, and rows always operate on columns, never the other way round. Here is an example of multiplying two matrices:

$$\overset{A}{\begin{bmatrix} 4 & 3 & 5 \\ 1 & 3 & 2 \\ 7 & 2 & 9 \end{bmatrix}} \overset{B}{\begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 7 \\ 9 & 2 & 1 \end{bmatrix}} = \overset{AB}{\begin{bmatrix} 65 & 37 & 30 \\ 32 & 22 & 24 \\ 103 & 49 & 30 \end{bmatrix}}$$

Matrix A x Matrix B = Matrix AB

Each entry in the answer matrix on the right is obtained by multiplying each of the entries in a row of the first matrix with its corresponding entry in a column of the second, and then adding up these products. The highlighted entry 37 in matrix AB is in row 1 and column 2 of AB. To obtain it, you take the highlighted row 1 of matrix A and multiply it term by term with the highlighted column 2 of matrix B, then add up the results:

$$(4 \times 3) + (3 \times 5) + (5 \times 2) = 12 + 15 + 10 = 37$$

If you multiply the matrices A and B in the opposite order, the product matrix BA is quite different from AB. A and B do not commute:

$$\overset{B}{\begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 7 \\ 9 & 2 & 1 \end{bmatrix}} \overset{A}{\begin{bmatrix} 4 & 3 & 5 \\ 1 & 3 & 2 \\ 7 & 2 & 9 \end{bmatrix}} = \overset{BA}{\begin{bmatrix} 18 & 17 & 25 \\ 70 & 41 & 93 \\ 45 & 35 & 58 \end{bmatrix}}$$

Matrix B x Matrix A = Matrix BA

An important feature of matrix multiplication is that a whole row of one matrix and a whole column of another must interact to produce a single cell in the product matrix. There is a passage of information from the many to the one and from the one to the many. I have used 3 by 3 matrices in the example above, but generally there is no limit to how many cells there can be in each row and column, and quantum mechanics makes use of matrices of infinite dimension, as well as matrices of fixed dimension. In Feynman's 'sum over histories' version of quantum mechanics, all possibilities are explored by nature before a definite outcome is chosen. The infinite possibilities of matrices allow for this.

Although a matrix is an array of numbers, each matrix has an important single number associated with it called the determinant, which shows the overall scaling effect of the matrix. The determinant is found by multiplying, adding and subtracting cells in a certain order. If the determinant is zero, the matrix is called 'singular', and this reveals that two rows or two columns of the matrix are encoding the same information. Each row or column of a matrix is equivalent to an equation connecting the quantities in it. Matrices, and the linear transformations that they can represent, are essential tools throughout physics.

The Pauli Spin Matrices

Non-commuting numbers are often portrayed as a special peculiarity of quantum mechanics, but successive rotations in space about different axes are naturally non-commutative, the order in which you apply rotations to an object affects its final orientation. You can easily demonstrate this for yourself by rotating a book by 90° about three perpendicular axes x, y and z in succession, and noting its final orientation. If you put the book back as it started, and repeat the three rotations in the order y, x and z, the book's final orientation will not be the same as before.

$$\begin{array}{cccc} \sigma_z & \sigma_x & \sigma_y & 1 \\ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} & \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} & \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{array}$$

The Pauli Spin Matrices

The four Pauli spin matrices displayed above are fundamental in quantum mechanics. They describe a quantum particle's spin and rotations about the x, y and z axes. The matrix on the right is just the unit matrix which leaves everything unchanged. When a matrix is applied twice in succession, it is multiplied by itself, and if you apply the rules of matrix multiplication described above, you will find that any one of the Pauli matrices when squared just gives the unit matrix: $\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1$. A product of two Pauli matrices representing different spatial directions just gives i times the matrix for the third direction, e.g. $\sigma_x\sigma_y = i\sigma_z$. These products are also non-commutative, their multiplication order matters, so for example, $\sigma_x\sigma_y = -\sigma_y\sigma_x$. The Pauli matrices, as well as being used to track particle rotations and spin, can account for the behaviour of any system at all that has only two possible states, like a bit that can represent 0 or 1, and these come up frequently in physics.

Pythagoras

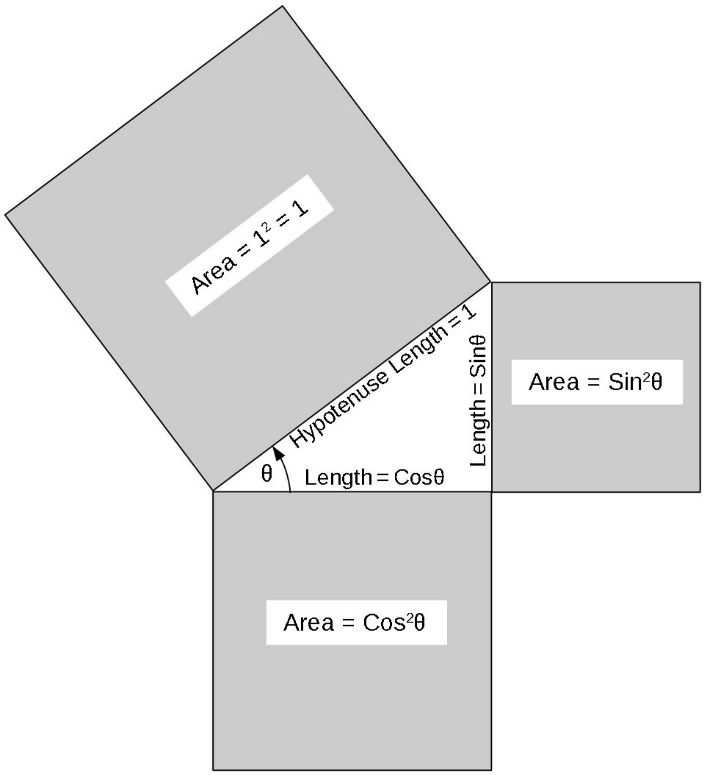


Fig 4.6 Pythagoras as addition of areas: $\mathrm{Sin}^2\theta + \mathrm{Cos}^2\theta = 1$

An animation of this constant sum of areas as the angle θ changes can be seen here:

https://youtu.be/7qO4IN0yUn8

Most people learn the Pythagoras theorem in school — the square on the hypotenuse is equal to the sum of the squares on the other two sides — but few people realise how central this theorem is to the deeper levels of mathematics and physics. Fig. 4.6 shows the Pythagorean relationship in terms of the areas on the sides of a right-angled triangle, and also the same relationship in the language of trigonometry, $\cos^2\theta + \sin^2\theta = 1$, relating the sine and cosine of an angle.

As the angle θ changes, sweeping around in a circle, the two smaller areas change size, but their sum, the area on the hypotenuse, remains constant. If we take the hypotenuse area as representing a constant amount of information that can be shared between two channels, the rotation of θ redistributes this information between the $Cos^2θ$ and $Sin^2θ$ channels. According to quantum mechanics, this process is happening continuously in all matter, but the information in the imaginary $Sin^2θ$ channel does not manifest itself in the reality of the outer world.

Pythagoras in 3D

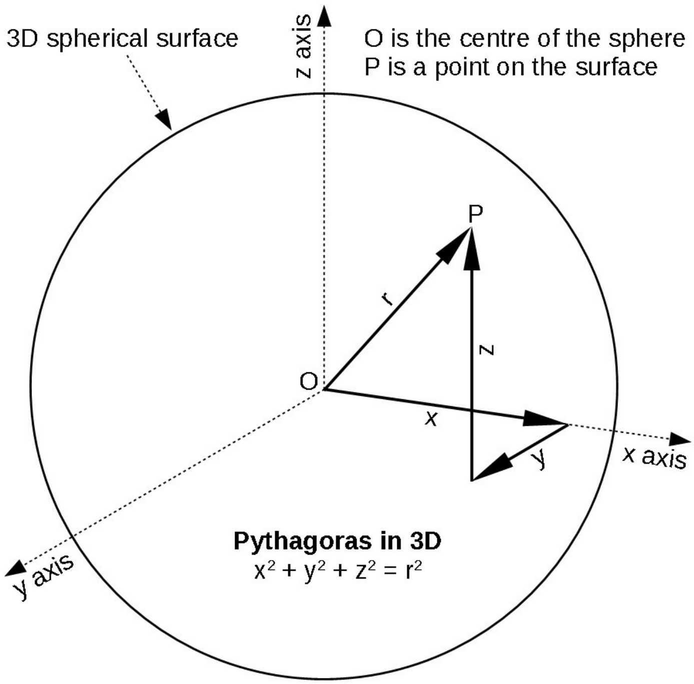


Fig. 4.7 Pythagoras in 3 dimensions

Another way of writing the Pythagoras theorem is

$$x^2 + y^2 = r^2$$

where r is the length of the hypotenuse, and the x and the y are the lengths of the other two sides that we previously described as Cosθ and Sinθ. If we add a third dimension, z to the x and y we get

$$x^2 + y^2 + z^2 = r^2$$

which is the 3 dimensional version of Pythagoras shown in Fig. 4.7, where r is the radius of the sphere.

Minkowski's 4D Space-Time

> . . . the very idea of space-time itself came from Minkowski who wrote, in 1908, 'Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.' In my opinion, the theory of special relativity was not yet complete, despite the wonderful physical insights of Einstein and the profound contributions of Lorentz and Poincaré, until Minkowski provided his fundamental and revolutionary viewpoint: space-time.
> Roger Penrose, *The Road to Reality*

Minkowski space-time links space and time with the Pythagorean type formula

$$x^2 + y^2 + z^2 - t^2 = r^2$$

Here, x, y and z are the three space dimensions and t represents time. The curious way in which the squares combine, + + + - , is called the Minkowski signature of space-time. But why is the t^2 term negative? This is a great mystery, and it is this change of sign that creates hyperbolic, rather than Euclidean geometry, and gives rise to the weird effects of special relativity — shrinking rulers, slowing clocks and twins of different ages. Some physicists have tried to work with this by making time imaginary, bringing in i, the $\sqrt{-1}$, and changing t to it so that $(it)^2$ becomes negative, $-t^2$. Whatever the correct interpretation may be, it does seem clear that time, when squared, is not just orthogonal to the normal space dimensions, but works in the opposite sense to them. The squares of the three space dimensions all add up to increase the radius of the bubble surface in Fig. 4.7, but the time coordinate squared works to reduce the bubble radius, it has an inward tendency.

Riemannian Geometry

The Pythagorean theorem has yet more to give, it is at the very foundation of geometry, as Bernhard Riemann showed when he used it to provide an invariant unit of measure for bent, non-Euclidian, spaces. (Note that here Einstein uses the notation x_1, x_2, x_3 instead of x, y, z, and x_4 instead of ct, where c is the speed of light).

> We have, . . . , good ground for the assumption that the "field-free", Minkowski-space represents a special case possible in natural law, in fact, the simplest conceivable special case. With respect to its metrical character, such a space is characterised by the fact that $dx_1^2 + dx_2^2 + dx_3^2$ is the square of the spatial separation, measured with a unit gauge, of two infinitesimally neigh-

bouring points of a three-dimensional "space-like" cross section (Pythagorean theorem), whereas dx_4, is the temporal separation, measured with a suitable time gauge, of two events with common (x_1, x_2, x_3). All this simply means that an objective metrical significance is attached to the quantity

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - dx_4^2$$

as is readily shown with the aid of the Lorentz transformations. Mathematically, this fact corresponds to the condition that ds^2 is invariant with respect to Lorentz transformations.

Albert Einstein, *Relativity*

Einstein introduces here the invariant quantity ds^2, the fundamental 'line element' of the Riemannian geometry that he used to formulate general relativity. The invariance of the infinitesimal line element ds^2 is fundamental in maintaining the proper relations between the different dimensions of space and time when they are bent. It guarantees that changes in one quantity are matched by compensating changes in another quantity, just as we have seen above in simpler examples of the Pythagorean relationship. The fact that general relativity is based on the changing coordinates of infinitesimally small differential quantities, means that its viewpoint is inherently tied to the ultra-small level, and it lacks a holistic overview. Not only did Einstein believe that interactions in the Universe are strictly local, but he also had total faith in the principle that everything is relative, and that there are no privileged reference systems. We know now that some form of instantaneous non-local synchronisation must take place in quantum entanglement that we look at in Chapter 11, and we will also see in Chapter 8 that a privileged reference system can be part of relativity. Physicists have struggled for over 90 years to marry quantum mechanics with general relativity, but without success.

5
Units of Measure

Continuous or Digital Reality?

Today's 'continuum' mathematics allows quantities to be arbitrarily large or small, but nature doesn't seem to work like that, it works in discrete chunks — quanta.

> . . . our real universe does not permit the unlimited chain of infallible operations, envisioned in continuum mathematics, and this has an influence on the ultimate nature of physical law.
> Rolf Landauer, *Information is Inevitably Physical*, in *Feynman and Computation*

> What if space-time is "discrete" that is composed of separate points, with no continuum between them? Feynman did like that idea because he felt that there might be something wrong with the old concept of continuous functions. How could there possibly be an infinite amount of information in any finite volume? Why do all those new particles appear when we try to put too much energy in one place? Could that be because there just isn't room for that much information?
> Marvin Minsky, *Feynman and the Cellular Vacuum*, in *Feynman and Computation*

Reciprocal Units

We can measure a length in kilometres, metres, millimetres, or any other standard length we chose, but in some applications it is more convenient to use reciprocal units — instead of measuring the length of something in metres, you can say how many somethings fit into a metre. Reciprocal units are widely used in solid state physics and crystallography. The arrangement of a unit 1 sitting between multiplication and division, governs the operation of the *uncertainty principle* in quantum mechanics, where momentum is measured by reciprocal length or *wave number* — the number of wave crests that fit into a length unit, and energy is measured by reciprocal time — the number of wave flips in one time unit. Any unit of measure implies its inverse: a reciprocal unit.

Even space can be inverted into *reciprocal space*. We will look at the great importance of Fourier duality in Chapter 10, and reciprocal space is just the Fourier transform of normal space. In solid state physics and crystallography, normal space is avoided as reciprocal space turns out to be far more convenient, the *direct lattice* of normal space being inverted in the origin to produce its dual *reciprocal lattice*. It is a property of Fourier duality that both representations contain the same information, just laid out in different forms. Like so many fields of physics, reciprocal space was first introduced by James Clerk Maxwell whose famous equations of electro-magnetism we look at in Chapter 7. To simplify complex engineering calculations of the forces in frameworks like girder bridges, Maxwell introduced reciprocal diagrams, in which the lines of force in the real structure, become polygons in the reciprocal diagram, and vice versa. I well recall the simplicity, but mind-bending nature of working with these diagrams as an engineering student.

Dimensional Analysis

Dimensional analysis is the name given to the technique of playing around with the units, or 'dimensions', used to represent quantities, without worrying about their particular values. For example, the speed of something must have the dimensions of length/time, L/T, irrespective of whether the length is measured in metres or inches, or the time is measured in seconds or years. The only three dimensions considered essential to physics are Mass, Length and Time, M, L and T, (Note: some physicists argue that electric Charge is also essential, and more fundamental than mass). Fig. 5.1 shows how other units of the S.I. system can be derived from the Kilogram (Mass), Metre (Length), and Second (Time). You can see immediately that the Kelvin measure of temperature and the Candela measure of light intensity are derived from other units, and that the Metre depends on the Second because of the fixed speed of light. In fact, convenience of use aside, many of these units could be scrapped if we wanted to get down to what is truly fundamental. Maxwell held that the trinity of Mass, Length and Time is fundamental, but do we really need as many as three fundamental dimensions? Arguments continue.

Dimensional analysis sometimes makes it possible to 'guess' how to write the equation that governs a process simply by playing around with combinations of M, L, and T, so that they balance on each side of an equation. For example, force has dimensions ML/T^2, mass has dimension M, and acceleration has dimensions L/T^2, so we might guess that we need mass times acceleration on the right of an equation to balance force on the left, $ML/T^2 = M \times L/T^2$. We have 'derived' Newton's second law of motion!

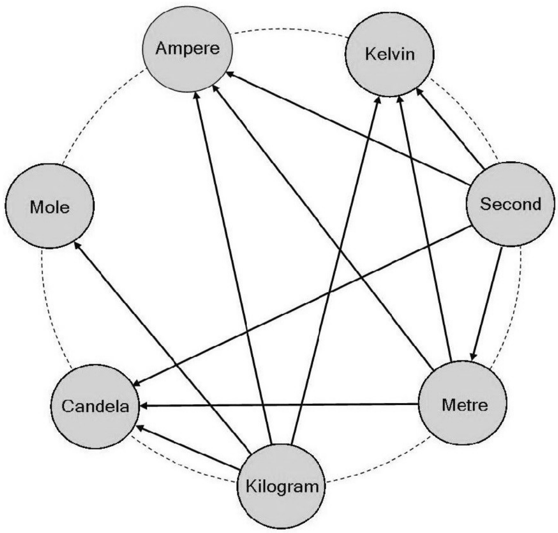


Fig 5.1 The seven S.I. units of measurement

There are special numbers called *dimensionless numbers* in which all the dimensions cancel out and you are left with no M, L or T dimensions. There are many examples of dimensionless numbers in science and engineering, and they often seem to exert a powerful control over processes, as the Mach number does for supersonic flight, or the Reynolds number does for fluid flow. One dimensionless number that especially fascinates physicists is the *fine structure constant* α, alpha, which sets the strength of the electromagnetic interaction between elementary charged particles. Its reciprocal has the value 137.0216..., and this is a great mystery — why should it be this particular pure number? We will see more about the role of the fine structure constant alpha in Chapter 13.

The Quantum Unit

The quantum of quantum mechanics is a unit, but a very special kind of unit with multiple dualities built into it, it is the unit of *action* known as Planck's constant, which physicists write as h, and I will often refer to as simply a Planck. The Planck is the standard digital currency of the Universe, and every physical change in the Universe requires one or more Plancks. But what is action? Nobody knows! But the *Principle of Least Action* is one of the most important ideas in physics, allowing all the major laws of physics to be derived from the principle that the Universe seeks the perfection of least action at every instant, just as Leibniz envisaged.

> Richard Feynman himself, even after having stalked action at closer distance than any other mortal through his theory of path integrals, would still say "I don't know what action is". Apparently there are more veils to be lifted.
>
> Tommaso Toffoli, *The Fungibility of Computation*, in *Feynman and Computation*

One clue to the role of action is that it shares the same ML^2/T dimensional composition as angular momentum, the rotational equivalent of the ML/T linear momentum possessed by a moving mass. The extra L arises because the angular momentum of rotation increases not only on with mass and speed, but also with the radial distance of the mass from the pivot point. From this perspective, the Planck transaction for each event in the Universe might be described as a simple twist of fate.

Fig. 5.2 shows the dimensional makeup of other important quantities in physics and their relationship with action. You can also see how force times time gives momentum, energy times time gives action, and so on. Mass, Length and Time are the essential quantities used by physics to describe the Universe, and quantum mechanics reveals that they all have an intrinsically *dual* nature. Position↔momentum and energy↔time are 'two in a bed' conjugate pairs governed by Heisenberg's uncertainty principle — the more room allocated to describing one of the pair, the less room there is for information about the other. Fig. 5.2 shows how a single Planck is a product of momentum and length, and how it is also a product of energy and time. Emmy Noether showed that it is the inherent symmetry in these dualities that allows the law of conservation of momentum to grant the freedom of relocation in space, and the law of conservation of energy to grant the freedom of when things happen in time. It is the multi-faceted nature of the Planck unit that allows the quantum mechanical wave function to encode many quantities at the same time, and furthermore, the Planck h, has a smaller twin, the 'reduced' Planck $\hbar$, pronounced 'h-bar', with the value h/2pi. Whereas h appears in the formula for the energy of a photon, E=hf, (Energy = Planck times frequency), it is the reduced Planck $\hbar$ that is the angular momentum unit for spin. Note that Fig. 5.2 does not show one other duality that can form a Planck: angular-momentum↔phase, where phase is a dimensionless number.

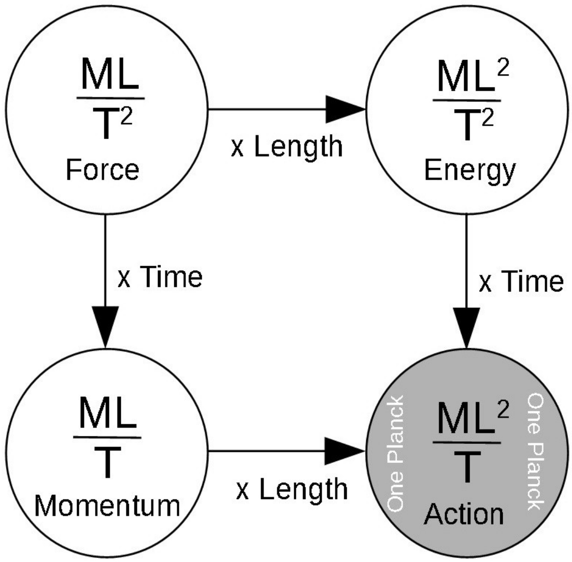


Fig. 5.2 Fundamental dimensions of physics.
M= Mass, L=Length, T=Time

Infinitesimals and Incommensurables

In his fascinating book *Infinitesimal*, Amir Alexander narrates the history of infinitesimals from the ancient Greeks, through struggles within the Roman Catholic church about their paradoxical nature and validity, up to their interpretation and use by Leibniz and Newton at the birth of calculus. As well as inventing the idea of atoms as the smallest units of material substance, the Greeks also postulated that a line is made up of a string of points, or 'indivisibles', which are the line's building blocks, and which cannot themselves be divided. But the trouble with this idea is that any positive magnitude that is allowed for an indivisible, however small, can always be divided again, but on the other hand, how can bits of line with zero length combine to make up a line with finite length? Disputes over this paradox continued for two millenia and were only officially settled long after the time of Newton and Leibniz, by the mathematical idea of limits. The modern form of calculus still relies on taking smaller and smaller units of measure *to the limit*, but does not set any size for this limit other than that it is greater than zero, yet smaller than anything you can think of — a fudge worthy of a politician. Infinitesimals turned out to be so useful in mathematics that they finally prevailed, despite having being denounced as heretical by the Catholic church.

Radial ↔ Rotary

The Greeks had also run into problems with other numbers apart from the infinitesimals, especially incommensurable numbers. If a square has a side of length 1, the diagonal has length $\sqrt{2}$, which is an irrational number, so there is no ratio of integers that describes the relationship between side and diagonal lengths, they are incommensurable.

The ratio pi of the circumference of a circle to its diameter appears every time you convert between angular units that measure round a circle, and linear units that go radially inwards and outwards. As pi is irrational, the measures of radius and circumference are also incommensurable. This does not matter for most purposes, but when a system is strictly digital like quantum mechanics with fixed size units, there must always be a slight discrepancy between rotary and radial measures, the two can never be in complete harmony because the Planck h and reduced Planck $\hbar$ differ by a factor of 2pi.

In Chapter 7 we will look at how Maxwell's equations describe the interaction of a pair of vector fields, the *scalar* electric field having the purely in-out property of *divergence*, and the vector magnetic field having the purely rotational *vector* property of *curl*. We will also see that a vital ingredient of the spinning cell model that Maxwell used to derive his famous equations is the requirement for the *surface speed* of each magnetic vortex cell to match the linear speed of the electric particles surrounding them, the electric particles can only roll on the surface of the cells, they cannot slip, thus ensuring that surface speeds of magnetic cell and electric particle are equal. This may seem trivial, but the importance of a shared surface speed between rotation and linear motion will appear again in the electron's zitter dance which we examine in Chapter 13.

A technique called *separation of variables* is the physicist's favourite mathematical trick for solving partial differential equations. It depends on being able to write an equation as the product of two separate parts. Both Laplace's equation that governs electrostatics and the Schrödinger equation of quantum mechanics can be solved in this remarkable way, each of them having a radial term that multiplies a rotational term. While physicists find this an almost miraculous property of the maths, and very useful, nobody can give a deep explanation of why it works, and why the radial↔rotary duality is so important in physics.

Certain or Uncertain?

The uncertainty principle is considered to be one of the central mysteries of quantum mechanics, yet just the same mathematics applies in everyday wave phenomena such as sound waves, which we at least have some chance of understanding. This was first made clear by Dennis Gabor, who later won the Nobel Prize for inventing holography, which depends on very similar mathematics. If you have used a guitar tuner, you will know that the tuner has to listen for a second or two after you pluck a string before homing in on what the frequency is, and the longer you allow it to listen, the more accurately it can identify the frequency. This is because the tuner measures the frequency of a sound by counting how many times the sound wave flips in a period of time; the shorter the time available, the less precise will be the frequency measurement. In the extreme case, if you shortened the time for measurement to less than one cycle of the wave, the frequency would be completely unmeasurable. There is also a flip-side to this acoustic uncertainty — the more accurately you measure the frequency of a note, the less you can pin down the moment in time when it was played.

A musical note has both a frequency *and* a duration. Whereas duration is measured in units of time such as seconds and has dimension T, frequency is measured in reciprocal time units such as cycles per second, with dimension 1/T. The same variable, *time*, is doing double duty, being used as T and as 1/T, and this inversion in the unit 1 is the root of the strange behaviour. The number 1 is a sort of pivot between the outer world of T, which extends from 1 to infinity, and the inner world of 1/T, which only runs from 1 down to zero.

Sound is recorded at high quality by measuring the air pressure 44,100 times every second. A digital sound file is simply a list of these sample values, with each sample recording what the air pressure was at that instant in time. But by the mathematical process of taking a Fourier transform, the same information can be represented as a list of frequency values, it is transformed from the *time domain* T to the *frequency domain*, 1/T. Using these complementary representations of the same data is routine in science and engineering, as each can reveal features that are hidden in the other domain. The frequency domain representation of a sound recording is like a recipe. Imagine that you have a sound synthesizer that can play many notes at the same time, each with a fixed frequency and amplitude. The frequency domain recipe specifies the amplitude of each note and its starting phase in its cycle, but says nothing about *when* anything happens in time. Yet, magically, if you set this multi-note synthesizer going, your original sound recording will faithfully unfold in time, the sound waves of the entire spectrum of frequencies interfere, boosting or cancelling each other at every moment in just the right way to recreate the original sound. The value at any *one instant* in the time domain is found by summing the contributions from *all the frequencies* in the frequency domain. Equally, the amplitude of a single frequency in the frequency domain can be obtained by summing the contributions from all the instants in the time domain. You can see why this requires the mathematics of matrices that we saw in Chapter 4. We will look further into Fourier duality in Chapter 10.

The dual Fourier domains of position↔momentum and energy↔time that occur in quantum mechanics are more complicated as they involve length and mass units as well as time. According to quantum mechanics, the wave function contains all the information that can be known about a quantum system, but different facets of this information are not available at the same time. You can ask the wave function of a particle about the particle’s position, or you can ask it about the particle’s momentum, but the two views are mutually exclusive, they live in opposite Fourier domains, and the more you know about one, the less you know about the other. To go from one view to the other you must perform a Fourier transform or inverse transform, and there is a little wriggle room of ‘uncertainty’ within the system when you switch domains. Mathematically, everything pivots around the quantum Planck unit. While the Planck is blamed for the Heisenberg uncertainty in products of quantities like position times momentum, this is unfair, it can also be described as the giver of certainty. If every dollar did not have the same value as every other dollar, what use would dollars be for transactions? The Universe’s single currency, the Planck, ensures consistency for all transactions in the Universe.

> For generations the public has been told that modern physics has changed our view of the universe. Our teachers tell our children that "In the world of classical mechanics, everything worked like clockwork, with deterministic certainty. But Quantum Theory has shown us that things are indeterminate. The mechanical, deterministic world of Newton has been replaced by one in which everything is uncertain and unpredictable." This view has become popular — but it actually puts things upside down. Uncertainty lay in the classical view — and it was quantum theory that actually showed why things could be depended on. It is true that Newton's laws were replaced by a scheme in which such quantities as place and time are separately indeterminate. But the implications of this are not what they seem — but almost the opposite. For it was the planetary orbits of classical mechanics that were truly undependable, because of chaotic interactions. In contrast, the "orbits" of electrons in atoms,

according to quantum mechanics, are extremely stable — and it is these that enable us to have certainty!
Marvin Minsky, *Feynman and Computation*

Duality

Science likes to divide and conquer. By simplifying processes to basic elements, they become easier to understand and describe mathematically. But many physicists have warned that with too much reductionism you risk losing sight of more holistic factors that may be vital to proper understanding. An electron is a charged particle that is the source of its electric field. It also acts like a bar magnet, threading a magnetic field through its North-South dipole. Fields were dreamt up by Faraday, and made mathematical by Maxwell, but can they be treated separately from their source particles with all honesty? We can also be misled by appearances — what seems to be an electric field to a stationary observer can appear like a magnetic field to a moving one — we look at this in Chapter 7. It was considerations about the implications of Maxwell's electro-magnetic equations, and their conflict with Newton's laws, that led Einstein to *special relativity*, a theory which is famous for clocks that slow and shrink when observed by someone in motion relative to them. Who is to say who has the correct viewpoint? It seems that a more holistic overview is required, but as we will see throughout this book, *duality* is everywhere in physics.

In the book *On Space and Time*, Shahn Majid presents many arguments to support his view that duality of representations is an essential requirement in physics, and he proposes the following postulate:

> **First principle of self-duality:** A fundamental theory of physics is incomplete unless self-dual in a representation-theoretical sense. If a phenomenon is physically possible then so is its observer-observed reversed one.

He goes on to propose what he calls a 'meta-equation' for the structure of physics:

> **Second principle of self-duality:** The search for a fundamental theory of physics *is* the search for self-dual structures in a representation-theoretic sense.

Majid also suggests that the desire of physicists to match up the dual views "creates a kind of 'engine' that could be viewed as driving the evolution of physics". In the current book, I am proposing that the Universe evolves through the interplay of dual, inner and outer, representations of the Universe. Fig. 5.3 is adapted from

Majid's original, and shows how the mass of an object constrains the scale at which it can exist in the Universe, mass increasing across to the right, and scale increasing upwards.

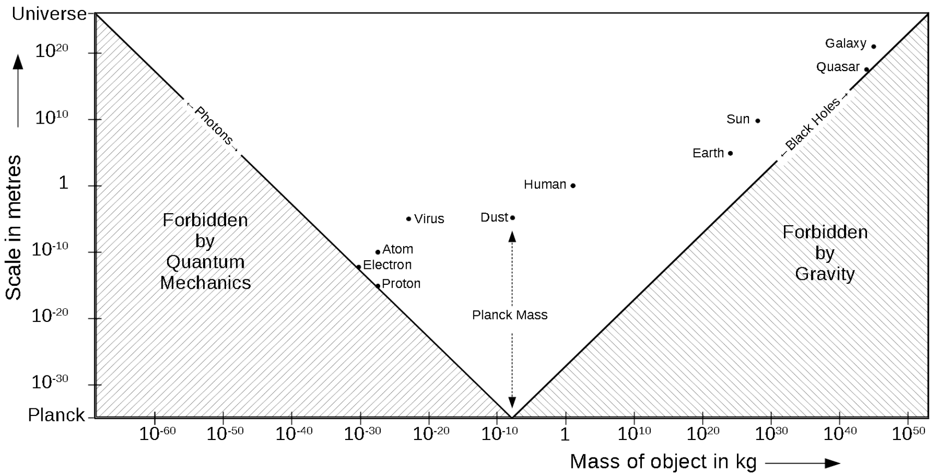


Fig. 5.3

The Universe and its contents as a log plot of Scale v Mass (from Majid)

The area to the left is forbidden by quantum mechanics — as you make a rest-mass carrying object smaller, the uncertainty principle ensures that it moves faster, and the electron and proton sit on the boundary line because this marks the scale at which they must reach the speed of light — this is called their Compton radius. The boundary line is marked 'Photons', because although a photon has no rest-mass, it has an energy equivalent to mass — thanks to $E=mc^2$ — that is inversely proportional to its wave-length, so all photons must live on this line, along with the electron and proton. The area to the right is bounded by the line marked 'Black Holes' that shows the limit of mass that space can accommodate at any scale, without turning into a black hole.

The pair of constraints on what can exist in the Universe displayed in Fig. 5.3 are the same ones used in Y. Jack NG's derivation of fundamental limits for clocks that we look at in Chapter 12. The diagram seems to suggest that the Universe is somehow mirrored above and below the Planck mass, constrained by quantum mechanics below, and gravity above.

The Einstein equation of gravity, which is central to general relativity, was derived by David Hilbert using the principle of least action at about the same time as Einstein got there by a less elegant route. We look at the principle of least action in

Chapter 6. The problem of marrying up the 20th century's two greatest theories, quantum mechanics and general relativity, is still the biggest unsolved problem in physics. In his contribution to the book *On Space and Time*, Fields medallist Alain Connes, finds a possible way by using non-commutative geometry, bringing in another duality, and arguing that matter possesses a special geometry which gives to space-time a fine structure at small scales in the form of a two-sided space he calls F. Connes writes the total action S as the sum of two parts S_{EH}, the Einstein-Hilbert action of empty space, and S_{SM}, which encodes what is known so far about the great array of particles in the standard model of physics, and in principle describes all the matter and particles in the Universe with the *exception* of gravity. He describes the S_{SM} action as "quite complicated", the formula taking four hours to typeset, and containing "twenty or more parameters put in by hand according to the experimental data, such as the masses of the various particles".

> "The additional term S_{SM} exhibits the fine texture of the geometry of space-time. This fine texture appears as the product of the ordinary 4-dimensional continuum by a very specific finite discrete space F. Just to get a mental picture one may, in first approximation, think of F as a space consisting of two points. The product space then appears as a 4-dimensional continuum with 'two sides'. … after a judicial choice of F, one obtains the full action functional, $S = S_{EH} + S_{SM,}$ as describing *pure gravity* on the product space M x F."
> Alain Connes, in *On Space and Time*

This mathematical description of 4D space-time as a 2-sided duality seems to be in agreement with the arguments in this book.

6
Perfection

Perfection

The idea that God and his works are perfect is a consensus belief of all religions, and has been the basis for many theological discussions over the centuries. Many of the theories of physics can also be derived from the idea that Nature chooses the most perfect outcome from the spread of possible ones. Wilhelm Leibniz invented the calculus at about the same time as Newton, and we still use his notation of dy/dx for the differential rate of change of y with x, and ∫ydx for the integral (sum) of y over many infinitesimal steps of x. Leibniz believed that the world is the most perfect of all possible worlds, and actualises every genuine possibility, it is only our finite experience of eternity that makes us doubt nature's perfection. To many people this has seemed ridiculous. How could a world filled with so much suffering be perfect?

> The actual flux presents itself with the character of being merely 'given'. It does not disclose any peculiar character of 'perfection'. On the contrary, the imperfection of the world is the theme of every religion which offers a way of escape, and of every sceptic who deplores the prevailing assumption. The Leibnizian theory of the 'best of possible worlds' is an audacious fudge produced in order to save the face of a Creator constructed by contemporary, and antecedent, theologians.
>
> A.N.Whitehead, *Process and Reality*, Gifford Lectures, 1927-8

Leibniz’s *Principle of Sufficient Reason* says that: "There must be a sufficient reason for anything to exist, for any event to occur, for any truth to obtain". It is also a pillar of eastern spirituality, and underpins all theories of karma. It was also a pillar of science until the 20th century, when physics abandoned it and adopted its own irrational belief in the god of randomness, despite the fact that perfect randomness is impossible, and quantum mechanics itself forbids the destruction of information. Unfortunately, it is very easy to confuse dilution of information with destruction of information. Our conventional computers appear to destroy information, two or more bits go into a logic gate like AND, but only one comes out. This is not a reversible process. But the information is not destroyed, it emerges in the waste heat that all our computers produce. For many years it was the received wisdom in physics that computation required a minimum waste of energy, rather like a heat engine, but it finally emerged that reversible computation is possible when using equal numbers of bits going in and out of a logic gate, and that this need not theoretically dissipate heat. If you have wondered how an atom can compute without dissipating heat, this is the answer, it uses reversible computation, recycling all the information. We will look at this in Chapter 9, and more detail can be found in Feynman's *Lectures on Computation*.

Perfection is not an easy concept to pin down, it is so subjective. When a cat catches a mouse, it is perfect for the cat, but far from perfect from the mouse's viewpoint. The ongoing game of predator versus prey is the engine of evolution that drives both predator and prey towards greater perfection. It is only possible to believe that the world is perfect if you can accept that everything happens for the greater good of all creation, whatever the consequences for the individual. The 'saintly' view is also that all suffering is ultimately to be learned from. Another take on perfection would be to ask, "what is the perfect story?" The perfect ending may be, "everyone lived happily ever after", but this is hardly a story. The ideal drama script needs the contrast of good and bad to keep us entertained, and challenges for humans to stand up to. Maximising the perfection of entertainment provided for occupants of the Universe may well be one of its design principles — the ancient Greeks and Romans seemed to believe that mortals are here to keep the immortals entertained.

If you imagine having God's tasks of creating and running a Universe, how would you do it better? You might think that you would like to abolish death, suffering, pain, hatred, creatures killing each other, etc. But it is hard to conceive of a better set of rules than those that we have to live by under Nature, any improvement you might think of always has a downside. If you want tighter controls to prevent creature abusing creature, then free will disappears. We would complain if we did not have free will, but we don't like it when someone else uses theirs to harm us. If creature does not eat creature, there is no recycling and the world is not sustainable. Without the continual interplay of creature and ecosystem there is no evolution of the exquisite forms that Nature develops. As a final excuse for the trials of human life, the Perennial Philosophy, like all the religions it draws from, offers a means of escape from the troubles of Earth, into a blissful acquaintance with God.

> One can only fully accept the blind necessity which rules the universe by holding closely through love to the God who transcends the universe.
> Simone Weil, *Gravity and Grace*

Truth

There is a close connection between truth and perfection. Gandhi's belief system was centred on *satya*, truth, and the mantra *Satyameva Jayate*, 'truth alone prevails', from the Mundaka Upanishad, was adopted as the motto of the newly independent nation of India in 1950. Gandhi wrote about his own understanding of 'truth alone prevails':

> The world rests upon the bedrock of satya or truth. Asatya, meaning untruth, also means non-existent, and satya or truth also means that which is. If untruth does not so much as exist, its victory is out of the question. And truth, being that which is, can never be destroyed. This is the doctrine of Satyagraha in a nutshell.
>
> Mahatma Gandhi, *Satyagraha in South Africa*, quoted in *The Upanishads*, by Eknath Easwaran

Quantum mechanics can only predict statistical probabilities for the outcomes of processes, but once an outcome is measured in the real world, it becomes an element of concrete reality, what Whitehead describes in his *process philosophy* as a concrescence, a new unit of unchangeable truth.

Creativity

The term Creative Design has been adopted by bible worshippers who deny mountains of scientific evidence to claim that the world was created a few thousand years ago, and try to ban the teaching of Darwinian evolution. This is unfortunate because creative design is an essential part of the Perennial Philosophy, which maintains that everything emerges from a single creative source, but this does not require an irrational interpretation of history, science, or anything else. You can find justification for almost any belief in the Bible, just as you can in the Koran, and equally, you can also find justification for the opposite belief. In the Cow chapter of the Koran, the prophet Mohammed dictated "There shall be no compulsion in religion", but this is conveniently ignored by Islamic militants, just as militant Christians have always ignored the total non-violence taught by Jesus.

> Once created or divinely informed, the universe has to be sustained. The necessity for a continuous re-creation of the world becomes manifest, according to Descartes, 'when we consider the nature of time, or the duration of things; for this is of such a kind that its parts are not mutually dependent and never co-existent; and, accordingly, from the fact that we are now it does not neces-

sarily follow that we shall be a moment afterwards, unless some cause, viz. that which first produced us, shall, as it were, continually reproduce us, that is, conserve us.' Here we seem to have something analogous, on the cosmic level, to that physiological intelligence which, in men and the lower animals, unsleepingly performs the task of seeing that bodies behave as they should. Indeed, the physiological intelligence may plausibly be regarded as a special aspect of the general re-creating Logos. In Chinese phraseology it is the Tao as it manifests itself on the level of living bodies.
Aldous Huxley, *The Perennial Philosophy*

It is by long obedience and hard work that the artist comes to unforced spontaneity and consummate mastery. Knowing that he can never create anything on his own account, out of the top layers, so to speak, of his personal consciousness, he submits obediently to the workings of 'inspiration'; and knowing that the medium in which he works has its own self-nature, which must not be ignored or violently overridden, he makes himself its patient servant and, in this way, achieves perfect freedom of expression. But life is also an art, and the man who would become a consummate artist in living must follow, on all the levels of his being, the same procedure as that by which the painter or the sculptor or any other craftsman comes to his own more limited perfection.
Aldous Huxley, *The Perennial Philosophy*

Randomness

Randomness is defined as a lack of form or pattern, so the adjective 'random' means 'formless, patternless, and therefore unpredictable'.
Hans Christian von Baeyer, *Information: the New Language of Science*

Whether you regard some data as possessing or lacking meaningful order is a personal judgement, or perhaps, a matter of faith. Randomness is a slippery concept, and it is easy to make the jump from 'formless, patternless and therefore unpredictable' to the assumption that the data is meaningless. Perennial philosophers maintain that everything is connected and full of meaning, that every event happens for a reason, or perhaps the sum of an infinity of reasons, and the more you are aware of the interconnectedness of all things, the further you have progressed along the spiritual path. The atheist denies that there is a single authority from which all the coordinated meaning of the world derives and only recognises the meaningless version of randomness. In the 20th century, atheism became the official faith of physics, special relativity banned the connectedness of everything in a universal Now, and the dominant Copenhagen interpretation of quantum mechanics, rather than accepting that an effect can be the result of too many causes to understand, chose to abolish cause and effect altogether for the outcomes of quantum measurements.

> There seem to be two kinds of people in this world, who in their extreme forms are those who see everything in the world as caused and meaningful and those who believe that God plays dice and that truly random processes happen. The determinists find it difficult to accept the random nature of some events because their minds are repelled by chaos, the uncaused. Indeed, the natural instinct of most people is to seek security in determinism and to find 'reasons' for why things happen, no matter how farfetched the reason may be. Nothing brings out the distinction between the determinist and the acausalist so much as the confrontation with serious illness in another person or even oneself. The determinist will seek meaning and a cause for such an illness — somehow the person has 'sinned' to bring this affliction upon himself. In fact the illness may have no cause at all and, like a radioactive atomic decay, it just happens.
> Heinz Pagels, *The Cosmic Code*

Mathematicians have tried to define randomness in several ways and have come up with various definitions, but their definitions are still a little bit random. In his book *A New Kind of Science*, Stephen Wolfram lays out his belief that physics can be explained in terms of the operation of cellular automata, very simple types of computer that we will look at in Chapter 9. It is entertaining to see a mathematician's struggle in describing randomness, so here are some examples of Wolfram's usage:

> Apparently random, . . . completely random, . . . for practical purposes random, . . . has many features of randomness, . . . fairly uniform random behaviour, . . . in many respects random, . . . in many respects completely random, . . . in some ways random, . . . somewhat random, . . . quite random, . . . remarkably random, . . . perfectly random.
> Stephen Wolfram, *A New Kind of Science*

Sensitive information is encrypted before being sent over public channels to make it appear random to snoopers. Any computer program, however complex, and however powerful the computer it runs on, can only produce pseudo-random numbers. The multiple wheels of the German Enigma machines of the Second World War scrambled a message beyond human recognition, but what one machine can scramble, another can unscramble, and the machines built to do this were some of our earliest computers. If a code breaker has sufficient time and processing power, she can always find faint traces of patterns in the data that are left by any purely logical encoding process, however complex. These traces are the cracks that allow the code to be broken, and the sole task of Colossus, the first electronic computer, was teasing out such tell-tale patterns in messages that had been scrambled by the rotors of German cypher machines.

When you want perfect randomness, with no possibility of discerning an underlying pattern in the data, you have to turn to Nature, which alone can provide such perfect "disorder". It is no good using a computer program like the built in 'random number generator' provided by programming languages; this will appear only 'apparently random'. For optimum randomness, you have to create a string of random digits by sampling the noise in an electronic component, or patterns in radioactive decay, or some other natural process, governed by quantum mechanics, that reveals no pattern to disqualify it from the title of "perfectly random". Quantum measurements exhibit the most perfect form of randomness known. Nature alone uses the most perfect encryption.

> Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin. For, as has been pointed out several times, there is no such thing as a random number — there are only methods to produce random numbers, and a strict arithmetic procedure of course is not such a method.
> John von Neumann, quoted in *The Computer from Pascal to von Neumann*

Commonly used cryptographic systems like Pretty Good Privacy use the Public-key method in which a public key is used to encrypt a message and a private key is used to decrypt it. The public key is freely available to anyone, and is used by the sender of the message, while the private key is secret, and known only to the receiver of the message. The system works because two very large prime numbers can be multiplied together easily to make a very, very large number, but splitting the big number back into its two prime factors is very difficult, so current computers are unable to crack the encryption in any useful timescale. One reason for the excitement about quantum computers is that they are predicted to be able to factorise large numbers much faster than classical computers.

A strange feature of information theory is that the more random a message appears, the greater is its information content. Claude Shannon defined the information content of a message in bits, deliberately ignoring the meaning of a message as being "irrelevant to the engineering problem of transmitting information". Shannon information is a measure of surprise. A 1 bit message has 2 possibilities, 0 or 1. A 2 bit message has 4 possibilities, a 3 bit message has 8 possibilities, etc. A 10 bit message string gives you 1024 possibilities, but there is still no meaning until the message is interpreted by connecting it with other data, such as a look-up table containing a different sentence for each binary number. Where a Shannon bit string contains patterns of bits that are repeated, there is redundancy; that part of the message lacks surprise and can be compressed to reduce transmission and storage costs. Compressing a message makes it shorter, but also makes it appear more random. One definition of randomness for a message is that it cannot be compressed. Atheistic physicists follow Shannon in ignoring any underlying meaning in natural communications as “irrelevant to the problem” of reverse-engineering Nature.

Chaos is the name used by mathematicians to describe a very specific type of unpredictability: deterministic behaviour that is very sensitive to its initial conditions. Tiny variations at the start can lead to large changes in behaviour — the butterfly effect. Chaotic systems may appear disordered and random but would be completely predictable if the initial conditions were known with infinite precision. Because of our lack of detailed knowledge and computing power we jump to the usual conclusion that the system itself has no internal order.

As well as permitting better encryption, a high level of randomness is needed in many fields. A casino takes great care to ensure that a roulette wheel has as little bias as possible to give perfectly unpredictable outcomes, creating a level playing field for gamblers, except for the zero 'singularity' which ensures the casino's profit in the long run. Opinion pollsters need a random distribution of samples to produce accurate forecasts. Monte Carlo methods in mathematics solve problems by injecting random numbers and observing the outcome. A simple example of this is measuring the area of an irregularly shaped picture hanging on a wall. If you were to throw a large number of darts to hit the wall randomly, the proportion of the total number of darts that landed on the picture would be the same as the proportion of the picture area to the total area of the wall.

A message that is more random is more surprising, you cannot predict it, and this is important for drama and entertainment also. If a plot does not surprise you, the drama is boring. Life is full of surprises.

The Law of Large Numbers

The *Law of Large Numbers* is a theorem of probability theory which says that the more times you perform the same experiment, the closer the average of the results obtained gets to the expected value. So for example, throwing a dice with face values 1,2,3,4,5,6, has an expected average value of 3½, and the more times you throw it, the closer the average will approach 3½. A casino may lose money for a few spins of a roulette wheel, but the law of large numbers guarantees that the casino will make a profit after a large number of spins.

The science of thermodynamics began in the 19th century with the work of physicists trying to improve the efficiency of steam engines. In his 1857 work *On the nature of the motion called heat*, Rudolf Clausius was the first to clearly state that heat is the average kinetic energy of molecules. Using this idea, James Clerk Maxwell derived the momentum distribution for the moving molecules, and his work was in turn extended by Ludwig Boltzmann, who applied probability theory and statistics to the enormously large numbers of molecules that make up even a small volume of gas. The law of large numbers ensures that the average properties of so many gas molecules will be very close to their expected values.

In the 20th century, Boltzmann's *statistical mechanics* became an important part of physics, being used wherever large numbers of degrees of freedom are present in a system. Statistical methods were pioneered by the first modern life insurance company, the *Society for Equitable Assurances on Lives and Survivorship*, which was established in 1762, and used newly developed mathematical and statistical tools to calculate premiums based on mortality rates. Actuaries study age-of-death statistics for large numbers of people in order to work out the probability of death at any particular age, and this information allows accurate insurance premiums to be calculated. Again, the law of large numbers smooths things out. But no actuary would say that a particular human whose death contributed to the statistics, died without any cause of death whatsoever. The Copenhagen interpretation of quantum mechanics, still the dominant theory amongst physicists, asserts that events occur without any cause but the God of randomness.

Whitehead's Perfect God

At the same time as quantum mechanics was emerging in the 1920's, Alfred North Whitehead developed his process philosophy as an update to the western philosophical tradition which he said was all 'footnotes to Plato'. He drew together concepts from religion, philosophy and science, and sought to correct the errors of other philosophers by linking the processes of being and becoming. Despite his criticism of Leibniz's 'maximally perfect world' idea in the quotation above, Whitehead described the primary action of God in his process philosophy as: ". . . the principle of concretion — the principle whereby there is a definite outcome from a situation otherwise riddled with ambiguity". This God of Whitehead's is a perfect reference which chooses the actual outcome of any event from amongst the inherent possibilities, and seems to be exactly what is missing from quantum mechanics which has only allowed the God of randomness to perform this function.

The Principle of Least Action

> Least Action Achieves All Actions
> Laozi, Dao De Jing, ca 500 B.C.

> Every Action in Nature takes place in the shortest way possible.
> Leonardo da Vinci

In around 1650, Pierre de Fermat showed that of all the possible paths that light might take to get from one point to another, it takes the path which requires the shortest time. This is called the *Principle of Least Time*, or Fermat's principle.

Fermat's principle of least time applies to a lifeguard who sees a person drowning in the sea somewhere along the beach from her position. The lifeguard can run faster than she can swim, so her best route is not the straight line between them, but a bent line that minimises the time taken to reach the person, running on the beach more, and swimming less. This is illustrated in Fig. 6.1.

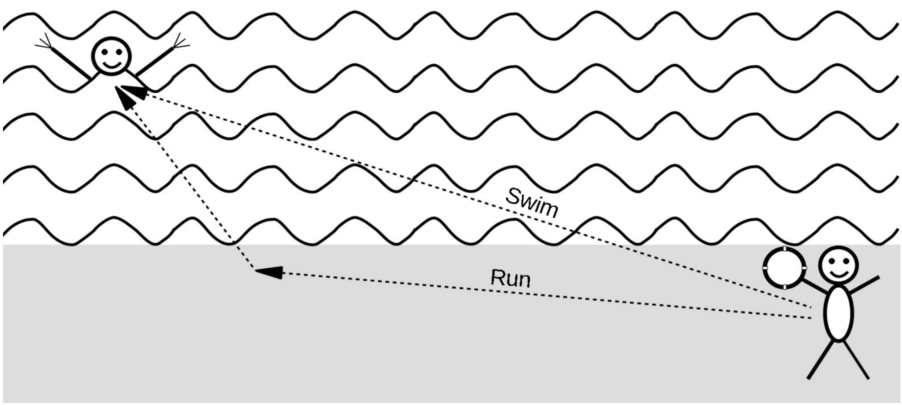


Fig. 6.1 Taking the fastest route

Light travels faster in air than it does in glass, so just the same 'refraction' of a ray of light is seen in optics as in the bending of the lifeguard's route. The law of refraction that optical lenses depend on can be derived from Fermat's principle.

There are also many other principles of perfection in physics which are facets of the more general *Principle of Least Action*, and all rely on minimising an action integral. Some examples are: *Gauss's Principle* of least constraint, *Hertz's Principle* of least curvature, *D'Alembert's Principle*, *Hamilton's Principle*, *Jacobi's Principle* and the *Euler-Lagrange Principle*. The book *The Variational Principles of Mechanics* by Cornelius Lanczos is the classic text on these least action principles, and Lanczos shows an almost religious passion for the subject.

The Principle of Least Action is the foundation of *analytical mechanics*, which offers a complete, and often more useful alternative to the Newtonian mechanics that is taught in schools. Each method gives the same answers, and one method can be derived from the other despite their very different methods. Analytical mechanics applies across all areas of physics including quantum mechanics, where Newtonian methods fail, and has a long history as the above quotations show. The modern form of analytical mechanics originated with Leibniz, Newton's rival, and has been developed and extended by a whole line of great mathematicians including Euler, Lagrange, and Hamilton. Somehow, as new physics comes along, it always seems to be at the heart of things, the mathematics is well developed, but nobody really understands why it works.

> We are taught to regard with awe the variational principles of mechanics. There is something miraculous about them, something timeless too: the storms of relativity and quantum mechanics have come and gone, but Hamilton's principle of least action shines among our most precious jewels.
>
> Tommaso Toffoli, *The Fungibility of Computation*, in *Feynman & Computation*

Analytical mechanics uses the *calculus of variations*, which is quite different from the standard calculus of Newton and Leibniz. Whereas the equations of standard calculus give a direct answer to a specific problem, the calculus of variations finds the true path that a process will follow by subtly varying the path so as to maintain a perfect balance between kinetic and potential energies at every step. This is the true path that Nature chooses. The calculus of variations subjects the equation that describes the *action* of a process to infinitesimal variations, looking for the perfect path where the action is unchanged by an infinitesimal variation to either side. Nature always chooses the path of least, or more correctly, stationary action. You could imagine a navigation app doing something similar when choosing the best route through a network of roads to take you from A to B.

One of the reasons that physicists like Toffoli see the workings of the least action principle as miraculous, is that Nature seems to see *every possibility*, and selects only the most perfect one for realisation. Another reason is that whereas Newtonian mechanics uses vectors which embody the concepts of directed force and momentum to calculate the motion of an object, analytical mechanics can produce the same answers working only with the object's kinetic energy and potential energy, and these are both scalar quantities that contain no information about directions at all!

> Analytical mechanics bases the entire study of equilibrium and motion on two fundamental scalar quantities, the "kinetic energy" and the "work function," the latter frequently replaceable by the "potential energy." . . . Since motion is by its very nature a directed phenomenon, it seems puzzling that two scalar quantities should be sufficient to determine the motion. . . And yet it is a fact that these two fundamental scalars contain the complete dynamics of even the most complicated material system, provided they are used as the basis of a principle rather than of an equation.
> Cornelius Lanczos, *The Variational Principles of Mechanics*

A major advantage of methods based on the least action principle is that they work throughout physics. Newtonian mechanics relies heavily on Newton's second law of motion which says that force equals mass times acceleration. But whereas Newtonian mechanics is contradicted by special and general relativity, the least action principle still holds, and was actually used by David Hilbert to derive Einstein's field equation of general relativity a few days before Einstein got there himself. Hilbert, besides being the foremost mathematician of his time, was gentlemanly enough to concede the glory to Einstein, whose ideas he had relied on anyway in his derivation.

In the inner quantum realm, at around the atomic scale and below, the inherent wavelike properties of particles become more important than their familiar, solid, outer world properties. What does the force on a wave mean? What rules the roost in the quantum world is energy, and this is just what the least action principle keeps account of. As we saw in Chapter 5, the unit of action, the Planck, is a product of energy times time, and the fixed size of the unit means that time is very long when energy is small, and time is very short when energy is great, the Planck has an inherent stretchiness in either direction.

In 1925 and 1926, Werner Heisenberg and Erwin Schrödinger formulated matrix mechanics and wave mechanics, two distinct but equivalent versions of quantum mechanics. Richard Feynman introduced a third version in his 1942 doctoral thesis titled "The Principle of Least Action in Quantum Mechanics". He later described how he had become entranced, like so many other mathematicians and physicists, by the least action principle:

> When I was in high school, my physics teacher — whose name was Mr. Bader — called me down one day after physics class and said, 'You look bored; I want to tell you something interesting.' Then he told me something which I found absolutely fascinating, and have, since then, always found fascinating. Every time the subject comes up, I work on it.
> *The Feynman Lectures on Physics* (1965), Vol. 2, 19.

The following simple example of the use of the least action principle is based on one from the Feynman lectures on physics.

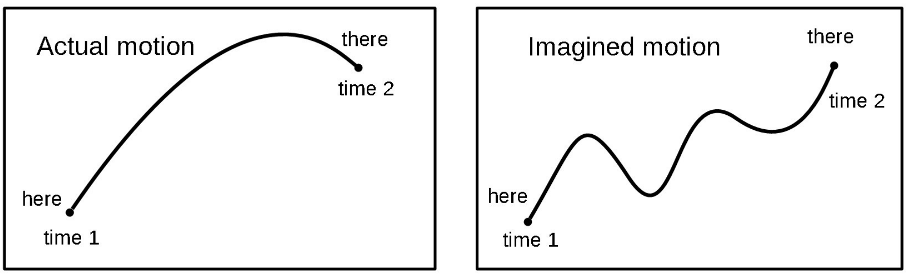


Fig. 6.2 Feynman's illustration

Suppose you have a ball which you throw through the air from here to there. It goes from the original place to the final place, along the path shown in Fig. 6.2 on the left, in a certain amount of time. Now imagine an alternative route the ball might take to get from here to there in the same amount of time, like the path shown in Fig. 6.2 on the right. The principle of least action says that if you calculate the kinetic energy at every moment on a path, take away the potential energy, and add up (integrate) all these differences between the two energies over the time taken for the whole path, the number you get will be bigger for the alternative path than that for the actual motion. If you find this hard to believe, there are computer programs that will let you try out different paths and do all the arithmetic for you, but you will find that the perfection of least action always wins.

What is being added up here is energy times time, which as we saw in Chapter 5, has the units of action, so the actual path is the one with the least action. In this example the potential energy of the ball is simply proportional to its height above the ground. As the ball rises, kinetic energy from its vertical motion is transferred to its potential energy in the Earth's gravitational field. If the ball falls, energy transfers back from P.E. to K.E. and the ball's vertical speed goes up. The total of K.E. plus P.E. stays the same (ignoring air resistance), because energy is conserved.

The integral, or sum, of kinetic energy minus potential energy over infinitesimal time steps is called the Lagrangian function, which is very important as it controls much of the behaviour of particles and other things in physics. Equally important is the Hamiltonian function which adds the kinetic and potential energies instead of subtracting them. The Hamiltonian and Lagrangian both encode much the same information and are useful for different purposes.

Feynman's *sum over histories* version of quantum mechanics assumes that in the quantum world, Nature always explores *all possibilities*, and it is only when we make a measurement to get macroscopic outer world information that a single possibility must be selected. In the famous two slit experiment, a photon particle of light 'goes through' both of two separate slits provided you don't observe it. We will see in Chapter 12 how the idea of a privileged dimension of *scale* makes this behaviour easier to understand. Some physicists favour the idea of many worlds, vast numbers of alternative universes where all the possibilities that don't actually happen in our world do become real. But there is really no need for the extravagant number of invisible, alternative universes that this requires. The internal processing power of matter at inner quantum world scales is quite sufficient to represent all possibilities. Our outer world sits on top of the inner world in the scale dimension, and only manifests a precisely calculated and pruned version of all possibilities. This is what we call reality. Reality is truth.

Another extraordinary aspect of the principle of least action is that it seems to work in teleological fashion towards a future end. Whereas the differential equations of Newtonian mechanics describe what happens at a localised point in space and time, the action principle is not localised to a point, but involves integrals over extended intervals of time and space, and the initial and final states of a system are often fixed at the start. Nature seems to take the future into account in its decision making.

Before writing his PhD thesis on least action applied to quantum mechanics, Feynman had worked with his supervisor John Wheeler on the Wheeler–Feynman *ab-*

sorber theory, which interpreted electrodynamics from the assumption that because the field equations of the electromagnetic field are invariant under time reversal, the solutions must also be symmetrical under time reversal. This made sense because there is no obvious mathematical reason for time reversal symmetry breaking, and the consequent distinction between past and future. A time-reversal invariant theory is more logical and elegant. One of the motivations for the absorber theory was that it was supposed to get rid of problems due to a particle interacting with itself, but it was later found to continue to suffer from these problems. There is also a transactional interpretation of quantum mechanics in which at the moment a photon is emitted, a deal is struck with the particle somewhere in the future that will absorb it.

Diversity

We have seen that the Lagrangian function is normally just the kinetic energy minus the potential energy. The Hamiltonian, the other most important function in analytical mechanics, is normally the Total Energy, the Kinetic Energy plus the Potential Energy:

$$\text{Lagrangian} = \text{K.E.} - \text{P.E.}$$

$$\text{Hamiltonian} = \text{K.E.} + \text{P.E.}$$

The Action is found by summing the Lagrangian function over all the infinitely small time steps that a system takes, and the Action is minimised by ensuring that the Lagrangian function is kept as small as possible throughout all those steps.

The extraordinary diversity of lifeforms on planet Earth is often seen as part of the perfection of Nature. Tommaso Toffoli, the physicist who invented the Toffoli (or Controlled NOT) reversible logic gate, has argued that the operation of the principle of least action throughout Nature, which shows up mathematically in the minimisation of the Lagrangian function, has the effect of maximising the possible choices available to each process. Toffoli’s arguments are too complicated to give in detail here, but by using a model computer consisting of a chain, and just a few rules governing the orientation of each link in the chain as time passes step by step, he is able to see what minimising the action does to the two types of energy, kinetic and potential, which are represented by different configurations of the links of the chain.

> . . . just as entropy measures, on a log scale, the number of possible microscopic states consistent with a given macroscopic description, so I argue that

> action measures, again on a log scale, the number of possible microscopic laws consistent with a given macroscopic behaviour.
>
> If entropy measures in how many different states you could be in detail and still be substantially the same, then action measures how many different recipes you could follow in detail and still behave substantially the same.
>
> Tommaso Toffoli, *The Fungibility of Computation*, in *Feynman & Computation*

In Toffoli's interpretation, Nature is maximising the freedom available to every process at every step. Nature, in all its workings, seems to maximise the freedom of choice.

> Love is that which enables choice.
> Always stronger than fear.
> Always choose on the basis of love.
> Forrest Landry, *Inscription on Magic Flight Launch Box*

Equipartition of Energy

We know that the energy of heat likes to spread itself around; whether by conduction through matter, or radiation as electro-magnetic waves, Nature seeks to share heat energy among objects until they reach the same temperature. In 1859, inspired by the work of Clausius, Maxwell proposed intuitively that gas molecules share their energy equally between linear motion and rotary motion in an *equipartition of energy*. This intuition proved to be correct although his calculations contained errors, as was common with Maxwell despite his undoubted genius and high mathematical status as a *second wrangler*, the second best mathematics student of his year at Cambridge. Ludwig Boltzmann later generalised Maxwell's principle of equipartition of energy to require that the energy should be divided equally between *all* the independent components of motion in a system. We will see how Maxwell derived his equations of electro-magnetism in Chapter 7.

Simplicity

There is a principle of philosophy called *Occam's Razor* which says that a simpler explanation of something is more likely to be correct than a more complicated one. Occam's Razor is often cited by physicists to dismiss overly complicated theories. One reason that David Bohm's holistic version of quantum mechanics never found much favour with physicists was that it added another layer of complexity to the theory without identifying any experimentally observable differences to standard quantum mechanics. We will look at Bohm's quantum mechanics in Chapter 11.

Feynman won a share of the 1965 Nobel prize for physics for his contributions to the theory of quantum-electrodynamics. His book, Q.E.D., is a remarkably simple description of this complex theory. It contains no mathematics at all, just words and pictures of little rotating arrows that keep track of the phases of processes and describe everything. Feynman's gift for finding simple visualisations of complex processes also showed in his creation of Feynman diagrams, which are now an essential tool for particle physicists.

> …in the further development of science, we want more than just a formula. First we have an observation, then we have numbers that we measure, then we have a law that summarizes all the numbers. But the real *glory* of science is that *we can find a way of thinking* such that the law is *evident*.
> *The Feynman Lectures on Physics* (1965), Vol. 1, 26-3.

Simplicity is a form of perfection. Although Feynman was an excellent mathematician, he had a strong dislike for excess mathematics being used at the cost of reduced understanding. In the mid 1900s, James Clerk Maxwell apologised for the use of advanced mathematics in his papers deriving the famous Maxwell equations of electro-magnetism that we will look at in Chapter 7. In the 20th century, physics became even more mathematical, and thus incomprehensible to anyone except those with years of specialised training,

A striking aspect of computation is its essential simplicity, and we look at this in Chapter 9. Modern software allows computers to not only crunch numbers, but do all manner of other things, and often better and faster than humans with years of training. But all these tasks amount to no more in the processing chips, than patterns of 0s and 1s interacting through the fundamental types of logic gate, AND, OR, and NOT. However complex the task may appear at the surface level, on the underlying level of bits, it breaks down into just many repetitions of a few simple processes.

7
Electro-Magnetism

An Electric World

Of the four known forces of nature, the Weak and the Strong forces only operate at scales much smaller than an atom, within the nucleus itself. Of the remaining two forces, Gravity is around 10^{40} times weaker than the Electromagnetic force and can safely be ignored for most purposes unless there is something enormous like a planet to provide gravitational attraction. It is thus left to the fourth force, Electromagnetism, to dominate almost all of the effects in our macroscopic world. The electric force is generally much stronger than the magnetic force, but owing to the overall neutral balance of positive and negative electric charges in atoms and matter generally, we do not fully appreciate how strong electric forces really are.

> The ultimate basis of an interaction between the atoms is electrical. Since this force is so enormous, all the plusses and all minuses will normally come together in as intimate a combination as they can. All things, even ourselves, are made of fine-grained, strongly interacting plus and minus parts, all neatly balanced out. Once in a while, by accident, we may rub off a few minuses or a few plusses (usually it is easier to rub off minuses), and in those circumstances we find the force of electricity unbalanced, and we can then see the effects of these electrical attractions.
>
> To give an idea of how much stronger electricity is than gravitation, consider two grains of sand, a millimeter across, thirty meters apart. If the force between them were not balanced, if everything attracted everything else instead of likes repelling, so there was no cancellation, how much force would there be? There would be three million tons between the two!
> Richard Feynman, *The Feynman Lectures on Physics*, vol 1

James Clerk Maxwell

When James Clerk Maxwell started work on electro-magnetism in 1860, there were four main effects that researchers including Coulomb, Ampère and Faraday had established from experimental observation:

1) Electric charges attract or repel each other with a force inversely proportional to the square of the distance between them. Unlike charges attract and like charges repel.

2) Magnetic poles attract or repel in a similar way, but they always come in pairs; for every north pole there is a south pole, single monopoles are never found.

3) An electric current in a wire creates a magnetic field around the wire, the direction of the field depending on the direction of the current.

4) A current is induced in a loop of wire when a magnet moves towards it or away from it, the direction of the current depending on the direction of the movement.

There were two rival ways of looking at these effects. Most researchers at the time were working on theories involving action at a distance, assuming that nothing happened in the space between electric charges and magnetic poles. Michael Faraday, on the other hand, believed that electrical charges and magnets fill the space around them with a field of lines of force, which produce electrical and magnetic effects by interacting with the lines of force from other electrical and magnetic sources. If you look at the patterns that iron filings make when scattered on a piece of paper over a magnet in Fig. 7.1, the physical existence of lines of force seems palpable.

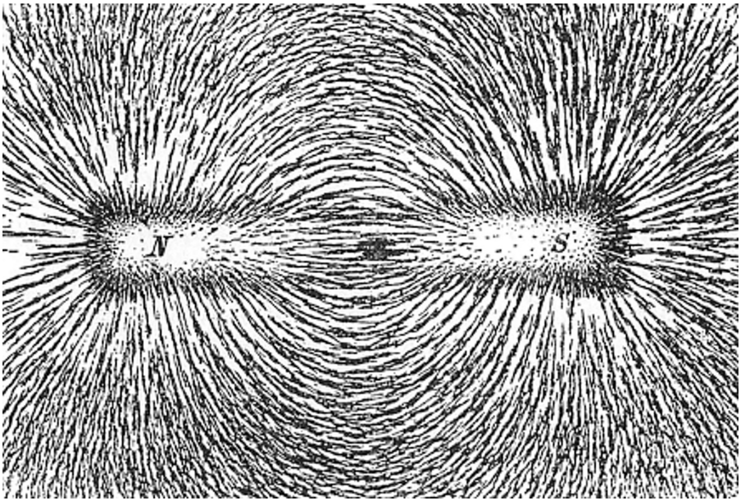


Fig. 7.1 Iron filings near a magnet

Newton, despite apparently invoking action at a distance with his theory of gravitation, had not liked its implication of non-local causality, which seemed like magic. Given the power of the church at that time, he was understandably careful to deny this heresy.

> That gravity should be innate, inherent, and essential to matter, so that one body can act on another at a distance, through a vacuum, without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity, that I believe no man, who has in philosophical matters a competent faculty of thinking, can ever fall into it.
> Isaac Newton, Letter to Richard Bentley

Maxwell didn't like the action at a distance idea either, he preferred Faraday's idea of an intermediate field between the interacting bodies to carry the energy and momentum from one place to another.

> . . . we cannot help thinking that in every place where we find these lines of force, some physical state or action must exist in sufficient energy to produce the actual phenomena.
> James Clerk Maxwell, *On Physical Lines of Force 1*, Phil. Magazine, 1861

Maxwell was a very good mathematician and used his skills to develop the first modern field theory, and turn Faraday's ideas on electrical and magnetic effects into equations. William Thompson (later Lord Kelvin) had already noted that the equations for the electrostatic force were of the same form as the equations for heat flow through a solid material.

> The laws of the conduction of heat in uniform media appear at first sight among the most different in their physical relations from those relating to attractions. The quantities which enter into them are temperature, flow of heat, conductivity. The word force is foreign to the subject. Yet we find that the mathematical laws of the uniform motion of heat in homogeneous media are identical with those of attractions varying inversely as the square of the distance. We have only to substitute source of heat for centre of attraction, flow of heat for accelerating effect of attraction at any point, and temperature for potential, and the solution of a problem in attractions is transformed into that of a problem in heat.
> James Clerk Maxwell, *On Faraday's Lines of Force 1*, Camb. Phil. Soc., 1856

Maxwell imagined that there is an incompressible fluid, of constant density, filling all of space. Faraday's lines of force were flows of this fluid, or flux. Pressure differences in the fluid caused by differences in electrical potential (voltage) or magnetic potential made the flux flow. Positive and negative electric charges were sources and sinks of the fluid. Forces depended on the speed of flow, or flux density of the fluid. This model was a start and could account for all the effects of static electric and magnetic fields and steady electric currents, but when anything was changing, he needed something more to account for the four effects listed above.

Spinning Cells

Maxwell managed to come up with an 'almost physical' model that could account for all the four effects above. He imagined that all of space is filled with tiny vortices, or spinning cells, the spin axis of each vortex being like a tiny bar magnet, with a North pole one end and a South pole the other. Each cell, because of its spin, exerts a centrifugal force against its surrounding cells, and if all the cells in a region spin in the same direction, the collective effect is an outward pressure at right angles to their spin direction.

The opposite would happen in the direction of the spin axis, the cells would try to contract, resulting in a tension, the incompressible fluid inside them compensating for the radial expansion. So the spin axes would behave like Faraday's lines of force, exerting an attraction along their length and a repulsion sideways. Faster spinning cells would produce greater attractive force along the lines and repulsive force sideways — a stronger magnetic field.

The spin axis of each vortex points along the direction of the magnetic field, and the direction of rotation determines whether you see a north or south magnetic pole when you look along the axis. The mass density of the cells can be so small that they offer no perceptible obstruction to ordinary matter. They only need to have some mass, and rotate very fast, to generate the necessary forces. Where the cells occupy the same space as ordinary matter, their behaviour is adjusted according to the magnetic susceptibility of the matter.

If the cells all spin in the same direction, there is a problem with the points of contact between the cells, where the surfaces would be moving in opposite directions. Maxwell solved this by placing even smaller particles between the cells to act as idler wheels, allowing the cells to rotate in the same direction. He further supposed that there would be no slippage between the surfaces of cells and idler wheels at points of contact, and that the idler wheels were particles carrying electric charge. Where the magnetic field was uniform, the electric particles would simply spin in situ between the cells. If there was an electromotive force, a voltage, across a region of cells, the electric particles would tend to be carried along the gaps between the cells without themselves rotating, forcing the cells around them to rotate as they went because of the lack of slippage. This behaviour corresponds to an electric current inducing a magnetic field, the effect 3 listed above. Electrical resistance would correspond to reduced freedom of the electric particles to move from cell to cell. Maxwell illustrated his ideas with a diagram which is reproduced in Fig. 7.2.

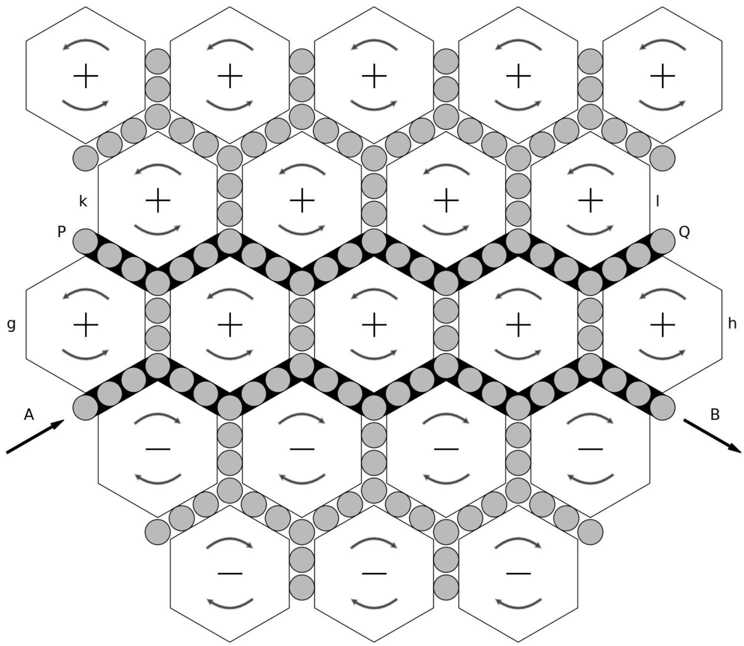


Fig. 7.2 Adapted from Maxwell's Plate V in his paper *On Physical Lines of Force 2*

An animation of this is available here: https://youtu.be/6GtBiWtSsTQ

The + or - in each spinning cell depends on the direction of rotation, and is equivalent to a north or south pole coming out of the page, it does not signify ± electric charge!

Maxwell described the mechanism of his model as follows:

Let A B represent a current of electricity in the direction from A to B. Let the large spaces [hexagons] above and below A B represent the vortices, and let the small circles separating the vortices represent the layers of particles placed between them, which in our hypothesis represent electricity.

Now let an electric current from left to right commence in A B. The row of vortices g h above AB will be set in motion in the opposite direction to that of a watch. (We shall call this direction +, and that of a watch -.) We shall suppose the row of vortices k l still at rest, then the layer of particles between these rows P Q will be acted on by the row g h on their lower sides, and will be at rest above. If they are free to move, they will rotate in the negative direction, and will at the same time move from right to left, or in the opposite direction from the current, and so form an induced electric current.

If this current is checked by the electrical resistance of the medium, the rotating particles will act upon the row of vortices k l, and make them revolve in the opposite direction till they arrive at such a velocity that the motion of the particles is reduced to that of a rotation, and the induced current disappears. If, now, the primary current A B be stopped, the vortices in the row g h will be checked, while those of the row k l still continue in rapid motion. The momentum of the vortices beyond the layer of particles P Q will tend to move them from left to right, that is, in the direction of the primary current; but if this motion is resisted by the medium, the motion of the vortices beyond P Q will be gradually destroyed.

It appears therefore that the phenomenon of induced currents are part of the process of communicating the rotatory velocity of the vortices from one part of the field to another.
James Clerk Maxwell, *On Physical Lines of Force 2*, 1861

Maxwell's cell and idle particle model was slightly non-physical — he justified his hexagonal depiction of circular vortices as just a convenient way of illustrating the model. But this model allowed him to write down the mathematical equations of mechanics that described it, and these corresponded to the known laws of electricity and magnetism. But there was still something lacking.

According to our theory, the particles which form the partitions between the cells constitute the matter of electricity. The motion of these particles constitutes an electric current; the tangential force with which the particles are pressed by the matter of the cells is electromotive force, and the pressure of

> the particles on each other corresponds to the tension or potential of the electricity.
>
> If we now explain the condition of a body with respect to the surrounding medium when it is said to be 'charged' with electricity, we shall have established a connexion between all the principal phenomena of electrical science.
> James Clerk Maxwell, *On Faraday's Lines of Force 3*, Cambridge Phil. Soc., 1861

To account for the effects of static electricity along with the other electromagnetic effects, Maxwell endowed his magnetic cells with enough elasticity for them to distort under electromotive force (electrical pressure or voltage) like springs. A voltage applied across a conductor causes a current to flow. Maxwell reasoned that, even in an insulator, a short lived displacement current would flow and distort the cells, just as a small movement is needed to compress a spring. Furthermore, the term he needed to add to one of the equations to account for this, would endow the whole set with a beautiful symmetry.

> James had shown how the electrical and magnetic forces which we experience could have their seat not in physical objects like magnets and wires but in energy stored in the space in and around bodies. Electrostatic energy was potential energy, like that of a spring; magnetic energy was rotational, like that in a flywheel, and both could exist in empty space. And these two forms of energy were immutably linked; a change in one was always accompanied by a change in the other. The model demonstrated how they acted together to produce all known electromagnetic phenomena.
>
> . . . All materials that have elasticity transmit waves. James' all-pervading collection of cells was elastic, so it must be capable of carrying waves. In an insulating material, or in empty space, a twitch in one row of idle wheels would be transmitted via their parent cells to the surrounding rows of idle wheels, then to the rows surrounding them, and so on. Because the cells have inertia they would not transmit the motion instantly but only after a short delay — the twitch would spread out as a ripple. So any change in the electric field would send a wave through all space.
>
> What is more, any twitch in a row of idle wheels would make the neighbouring cells turn a bit and so generate a twitch in the magnetic field along the cells' axes of spin. All changes in the electric field would therefore be accompanied by corresponding changes in the magnetic field, and vice versa. The

> waves would transmit changes in both fields; they were electromagnetic waves.
>
> Basil Mahon, *The Man Who Changed Everything*

Maxwell was now able to describe all the known effects of electro-magnetism mathematically, and found that the set of equations he had produced could be reexpressed as wave equations. Wave motion had already been studied extensively and the equations describing it were well known, so Maxwell was able to calculate the speed c of his waves from two constants that appeared in his equations to describe the elasticity and density of the medium, μ the magnetic constant, and ε the electric constant. Putting the experimentally measured values for these constants into the formula $c^2 = 1/\mu\varepsilon$, Maxwell found the speed of electromagnetic waves in air to be 193,088 miles per second. Just twelve years earlier, the speed of light had been measured to be 195,647 miles per second.

> The velocity of transverse undulations in our hypothetical medium, calculated from the electro-magnetic experiments of Kohlrausch and Weber, agrees so exactly with the velocity of light calculated from the optical experiments of M. Fizeau, that we can scarcely avoid the inference that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena.
>
> James Clerk Maxwell, *On Faraday's Lines of Force 3*, Cambridge Phil. Soc., 1861

> Imagine Maxwell's feelings when the differential equations he had formulated proved to him that electromagnetic fields spread in the form of polarised waves, and at the speed of light! To few men in the world has such an experience been vouchsafed . . . it took physicists some decades to grasp the full significance of Maxwell's discovery, so bold was the leap that his genius forced upon the conceptions of his fellow-workers.
>
> Albert Einstein, *Considerations Concerning the Fundamentals of Theoretical Physics*

When Maxwell died in 1879, few people understood or believed in Maxwell's work, but in 1887, after eight years of heroic experimental efforts, Hertz succeeded in transmitting electromagnetic waves across his laboratory — the radio age had begun.

Divergence and Curl

In modern form Maxwell's equations connect the vectors describing the magnetic and electric fields at a particular point in space. A well known example of a vector field is the scattering of little arrows describing the strength and direction of the wind on a weather map, each point having its own arrow. To understand the equations better, we need a brief description of divergence and curl, terms coined by Maxwell (although he used the term convergence, the opposite of divergence). The Helmholtz theorem assures us that a vector field is completely determined by just two quantities — its **DIV**ergence and its **CURL**.

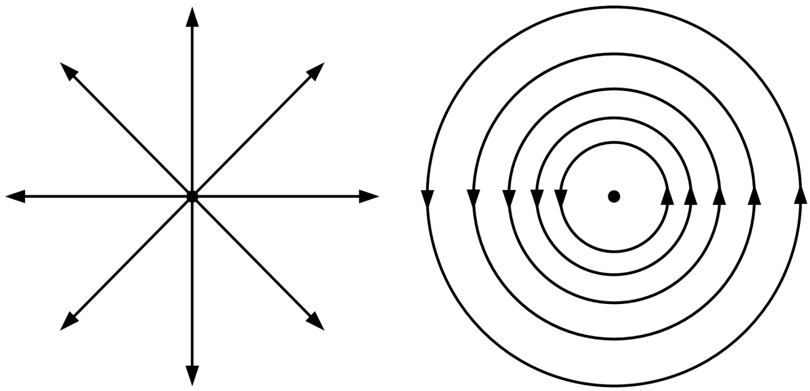
The divergence of a vector field is a measure of its expansionist or contractionist tendency in all three dimensions at the point in question. Rather than restricting itself to a single direction in 3D space, divergence is an effect belonging to the *scale dimension* that we look at in Chapter 12.

> Imagine standing at the edge of a pond. Sprinkle some sawdust or pine needles on the surface. If the material spreads out, you dropped it at a point of positive divergence; if it collects together, you dropped it at a point of negative divergence. . . A point of positive divergence is a source, or 'faucet'; a point of negative divergence is a sink, or 'drain'.
> David J Griffiths, *Introduction to Electrodynamics*

The curl of a vector field is a measure of how much it curls around the point in question, it is all about rotation.

> Imagine (again) that you are standing at the edge of a pond. Float a small paddlewheel (a cork or toothpicks pointing out radially would do); if it starts to rotate, then you placed it at a point of nonzero curl. A whirlpool would be a region of large curl.
> David J Griffiths, *Introduction to Electrodynamics*

In Fig. 7.3, the drawing on the left of shows how electric field lines and flux spread out as they DIVerge around a stationary point charge. The drawing on the right shows how magnetic field lines and flux CURL around a wire that carries an electric current of charged particles out through the page towards you.



Electrostatic field of a point charge. Magnetostatic field around a long wire.

Fig. 7.3 Electric and magnetic vector fields. The central dot on the right represents a wire carrying current out through the page.

When everything is stationary and the fields don't change with time:

> An electric field is pure divergence proportional to the electric charge enclosed, it has no curl.
>
> A magnetic field is pure curl, proportional to the electric current it surrounds, it has no divergence.
>
> Electric field lines originate on positive charges and terminate on negative ones.
>
> Magnetic field lines do not begin or end anywhere, as they have no divergence. They either form closed loops or extend to infinity.

Maxwell's Equations

Maxwell's equations are simplest when there is dependency on time, but space is empty, and there is no electric charge or current. With **E** representing the electric field, and **H** the magnetic field, Maxwell's equations can then be written like this:

1) $\text{div}\,\mathbf{E} = 0$

2) $\text{div}\,\mathbf{H} = 0$

3) $\text{curl}\,\mathbf{E} = -(1/c)\partial\mathbf{H}/\partial t$

4) $\text{curl}\,\mathbf{H} = (1/c)\partial\mathbf{E}/\partial t$

Maxwell's Equations
in empty space

I have used Gaussian units which highlight the role of the constant speed of light c as a kind of gear ratio between the electric field **E** and the magnetic field **H**, and $\partial\mathbf{H}/\partial t$ means the rate of change of **H** with time. These equations completely describe the electric and magnetic fields at the point in question. They say:

1. The electric field at the point has no divergence, so there is no electric charge present.
2. The magnetic field has no divergence — there are no separate magnetic sources or sinks, they come only in north/south pairs.
3. A magnetic field **H** changing in time wraps an electric field around it.
4. An electric field **E** changing in time wraps a magnetic field around it.

Together, the third and fourth equations describe how the fields work together to sustain wave motion.

> . . . the changes in the combined field of electrical and magnetic forces spread out in a kind of continuous leapfrogging action.
>
> . . . The **E** and **H** waves always travel together: neither can exist alone. They vibrate at right angles to each other and are always in phase.
>
> Thus any change in either the electrical or magnetic fields sends a combined transverse electromagnetic wave through space at a speed equal to the ratio of the electromagnetic and electrostatic units of charge.
> Basil Mahon, *The Man Who Changed Everything*

Maxwell's equations become slightly more complicated when electric charges are present. The div **E** of equation 1 is no longer zero, but describes the amount of electric charge present, and the curl **H** of equation 4 needs an extra term for the electric current added to the rate of change of **E** in time.

Maxwell's equations not only describe electro-magnetism and most of the workings of our outer world, but also carry relativity hidden inside them. It was consideration of Maxwell's equations and their apparent conflict with Newton's laws of motion that led Einstein to special relativity in 1905. Lorentz had already published his transformation equations for time and distance, with strange slowing of clocks, shrinking rulers, and mass increasing with velocity, and these effects were in conflict with the familiar Newtonian laws of motion, yet worked quite happily with Maxwell's equations. Einstein was also struck by the fact that what appears to be a magnetic field for one observer can appear to be an electric field to another observer in motion relative to the first, an effect that is explored in the Feynman Lectures II 13.6, with an analysis that relies on the fact that charge, unlike mass, length and time, is unaffected by relativistic motion because it is always conserved. An animation of Feynman's description can be seen here:

https://youtu.be/EQMN4u4jdcg

It later became clear that the Lorentz transformations and special relativity were all along implied by Maxwell's equations, they not only describe the laws of electro-magnetism, but the very structure of space-time itself.

Must Physics be so Abstract?

It was only thanks to his 'not quite realistic' spinning cell model of electrical and magnetic effects that Maxwell was able to write down equations for mechanical effects that corresponded exactly to the four electrical and mechanical effects observed by experimenters. But he then threw away the scaffolding of his mechanical analogy, rather embarrassed by its evident impracticality, and did not even mention it in his great *Treatise on Electricity and Magnetism* that was published in 1873. Nowadays we just use his equations without any physical insight, many students never even learning about the model he used to derive his equations. In his series of papers on electro-magnetism leading up to his equations, Maxwell emphasised the need for mental models in understanding phenomena, and even apologised for his use of so much advanced mathematics that put his work beyond the understanding of people without the necessary training, including Michael Faraday, the experimental genius who had supplied the concepts that Maxwell's work was based upon. It was Faraday who gave us the concept of a field to carry influences from one place to another rather than some sort of magical action at a distance that was the predominant idea at the time. In the 20th century, physics became even more mathematical, and this allowed great progress, but great physicists like Richard Feynman would still emphasise the need for physical intuition as well as equations — sometimes.

> A physical understanding is a completely unmathematical, imprecise, and inexact thing, but absolutely necessary for a physicist.
> Richard Feynman, *The Feynman Lectures on Physics*, vol 1

> Inspired by Faraday's geometrical musings, Maxwell created an electromagnetic universe that cannot be effectively reduced to mental images. All self-imagined analogs to visualize the field are in some way deficient. Yet the

> mathematical rendering of the field is complete and accurate. Maxwell likened the situation to that of a bell ringer who tugs ropes that dangle through holes in the ceiling of the belfry; the bells themselves and their actuating mechanism remain a mystery. Maxwell's contemporary, Heinrich Hertz, put it more bluntly: "Maxwell's theory is Maxwell's equations." Or in the words of Nobel prize-winning physicist Richard Feynman, nearly a century later, "Today we understand better that what counts are the equations themselves and not the model used to get them. We may only question whether the equations are true or false. This is answered by doing experiments, and untold numbers of experiments have confirmed Maxwell's equations. If we take away the scaffolding he used to build it, we find that Maxwell's beautiful edifice stands on its own. He brought together all of the laws of electricity and magnetism and made one complete and beautiful theory."
> Alan Hirshfeld, *The Electric Life of Michael Faraday*

In Maxwell's spinning cell model, the surface speed of the cells played a crucial role, being shared between rotation of the magnetic vortex, and linear motion of his electric idler particles. We will see a similar role for a shared surface speed reappearing in the zitter dance of electrons that we look at in Chapter 13.

The Aether

In 1873, at the very end of his 1000+ page *Treatise on Electricity and Magnetism*, which is probably the second most influential book in the history of physics, Maxwell returned to the action at a distance theories of his rivals and the necessity of an aether:

> If something is transmitted from one particle to another at a distance, what is its condition after it has left one particle and before it has reached the other? If this something is the potential energy of the two particles, as in Neumann's theory, how are we to conceive this energy as existing in a point of space, coinciding neither with the one particle nor with the other? In fact, whenever energy is transmitted from one body to another in time, there must exist a medium or substance in which the energy exists after it leaves one body and before it reaches the other, for energy, as Torricelli remarked, 'is a quintessence of so subtle a nature that it cannot be contained in any vessel except the inmost substance of material things.' Hence all these theories lead to the conception of a medium in which the propagation takes place, and if we admit this medium as a hypothesis, I think it ought to occupy a prominent place in

> our investigations, and that we ought to endeavour to construct a mental representation of all the details of its action, and this has been my constant aim in this treatise.
>
> James Clerk Maxwell, *A Treatise on Electricity and Magnetism 2*

To see if it would be possible to do an experiment to measure the 'drift' of the earth through the aether, Maxwell did some calculations and concluded that the sensitivity required was unattainable. After Maxwell's death Michelson took this as a challenge and, after eight years of thought, he built the first interferometer, an instrument capable of measuring distances with a precision of just one wavelength of light. The negative result of the famous Michelson-Morley experiment was a great surprise — no motion of the Earth through the aether was detected. To account for this negative result, FitzGerald proposed that any object moving through the aether would contract just enough to make the motion undetectable — shrink the ruler as well as the object, and you end up with the same measurement.

> We know that electric forces are affected by the motion of the electrified bodies relative to the ether and it seems a not improbable supposition that the molecular forces are affected by the motion and that the size of the body alters consequently.
>
> G F FitzGerald, letter to Science 1889, quoted by Harvey R Brown in *The Origins of Length Contraction 1: The FitzGerald-Lorentz Deformation Hypothesis*, 2001

His proposal made sense because it is electrical forces that hold matter together, and Oliver Heaviside had already used Maxwell's equations to predict something similar in 1888. FitzGerald also produced an equation showing that an object approaching the speed of light would be flattened like a pancake in the direction of travel. Lorentz independently came up with the same formula and applied it to slowing time as well as shrinking distance.

It is now generally accepted that with special relativity in 1905, Einstein abolished the aether along with the Now, but it may well be that we simply have not found a good model for it. We look at Lorentz's alternative to special relativity in Chapter 8. If you accept the premise of this book that all information in the Universe is represented in dual forms, mirrored at the atomic scale, a new information transport system becomes available. Torricelli's description of the aether that was quoted by Maxwell above, as 'a quintessence of so subtle a nature that it cannot be contained in any vessel except the inmost substance of material things' seems in harmony with this idea. In our outer 3D experience of the world, information uses particles moving through space as its means of transport, but if the dual version of this information can take a short cut through the inner sub-atomic world, no exterior transport system is necessary — no aether is required.

The behaviour of photons also supports this idea. A photon has no rest mass and is never directly observed, we can only infer its existence from changes it induces in the electron that gives birth to it, or absorbs it as it dies. If we can only infer the existence of photons from the behaviour of atoms, how do we know that a photon has really travelled through space and did not take a short cut through the sub-atomic information domain?

We know that a photon can travel by many routes simultaneously as long as it is not observed. In fact, before observation, the general path of a photon is more like a bubble expanding in three dimensions at the speed of light than a particle travelling in a one dimensional straight line. The photon only becomes located in space when the bubble is pricked by the electron that absorbs it, and then it is gone. This is one of the most baffling effects in physics. We look at it further in Chapter 11.

Symmetry

Something has symmetry if you can change it in some way without it appearing any different. Symmetry became an important topic in physics after 1918, when Emmy Noether published her theorem showing that each conservation law of physics also implies a symmetry. The conservation of energy is reflected in the symmetry that allows a physicist to measure the energy of something *whenever* they want and get the same result. The conservation of momentum is reflected in the freedom to measure something's momentum *wherever* you chose in space.

The electric and magnetic fields, **E** and **H**, of Maxwell's equations can each be expressed as the rate of change of a potential in space, an electric scalar potential, and a magnetic vector potential. We are familiar with voltage which expresses the *potential difference* between the two terminals of a battery. In most cases, it is only the potential difference that matters, you might be charged up to a high voltage potential relative to the ground but a battery powered device held in your hand would still behave normally. The freedom to change a potential without affecting outcomes is very useful in electromagnetic theory, and it became the prototype for the modern gauge theories of physics.

The most fundamental physical symmetry known to physics is Charge, Parity and Time symmetry. The CPT theorem is believed to apply to all physical processes. It says that a process will look the same if you flip all the three quantities of charge, parity and the direction of time. Charge just flips between + and -, parity reversal is like a mirror reflection, and time reversal is like watching a video play backwards, from finish to start. A +ve anti-electron, a positron, going backwards in time, and looked at in a mirror, appears to physicists just the same as a -ve electron moving forwards in time. Physics provides no explanation of why CPT reversal exists, but here again, this mystery makes sense under the duality between inner and outer information domains of the Universe that we are considering.

The in-out duality of charge in the Universe is striking when the scales of the Universe are portrayed with a common centre in the form of the AUM Mandala of the Introduction. Every atom has one or more negatively charged electrons orbiting around one or more positively charged protons in the nucleus, with a gap between them of 10^5 in the scale dimension, and recall that electrons and protons are the only stable particles that carry rest mass and charge.

If you look at a bubble surface from outside, you see a mirror image, or parity reversal of what you would see from inside the bubble, and the same must apply to an atom. The inner quantum world representation in the nucleus sees the atomic 'surface' behaviour of an orbiting electron with parity reversed, the dual worlds mirror each other.

In terms of the big bang model of cosmology, the centre of the AUM mandala can represent the big bang, and the direction of the arrow of time is outwards. Reversed time runs inward rather than outwards. In the sub-atomic world it is the reciprocal of time, frequency, that dominates, and it is impossible to say exactly when something occurs because of the Planck unit of wiggle room granted by the quantum mechanical uncertainty principle.

When time is measured by the rotations of the hands of a clock, you can reverse time by simply making them rotate in the opposite direction. In Chapter 10 we will look at Fourier duality and its dual time and frequency domains that can represent the same information in two different forms. The mathematics of Fourier Transforms requires positive and negative frequencies in the frequency domain representation, which correspond to two opposite directions of rotation. The phases which keep track of rotations in quantum mechanics are unobservable in the outer world but absolutely essential to its workings. Perhaps the inner world represents information in both forward and reverse time, all kept within a quantum Planck unit, and hidden away from our outer world.

8

Now

Now

To anyone but a physicist, the existence of a universal Now might seem obvious. Someone the other side of the Earth might have a very different time showing on their clock, but thanks to technology, you can speak to them almost in the Now, with only a small transmission delay. As the future does not yet exist, and the past is only accessible through evidence that exists in the present moment, there really is no time but the present moment — NOW. The timeless, eternal Now is a central feature of the Perennial Philosophy.

> "From Hobbes onwards, the enemies of the Perennial Philosophy have denied the existence of an eternal now. According to these thinkers, time and change are fundamental; there is no other reality. Moreover, future events are completely indeterminate, and even God can have no knowledge of them.
>
> ...The existence of the eternal now is sometimes denied on the ground that a temporal order cannot co-exist with another order which is non-temporal; and that it is impossible for a changing substance to be united with a changeless substance. This objection, it is obvious, would be valid if the non-temporal order were of a mechanical nature, or if the changeless substance were possessed of spatial and material qualities. But according to the Perennial Philosophy, the eternal now is a consciousness; the divine Ground is spirit; the being of Brahman is *chit*, or knowledge. That a temporal world should be known and, in being known, sustained and perpetually created by an eternal consciousness is an idea which contains nothing contradictory.
>
> ...The present moment is the only aperture through which the soul can pass out of time into eternity, through which grace can pass out of eternity into the soul, and through which charity can pass from one soul in time to another soul in time. That is why the Sufi and, along with him, every practising exponent of the Perennial Philosophy is, or tries to be, a son of time present."
> Aldous Huxley, *The Perennial Philosophy*.

Physicists generally deny the existence of a universal Now because simultaneity is relative, according to Einstein's theory of special relativity, for observers who are in relative motion. Nevertheless, physicists somewhat hypocritically invoke a cosmic shared Now when referring to the Big Bang, and a Now is also implied when they talk of a meaningful age that is shared by all parts of the Universe. If everyone's Nows are different, what could the age of the Universe mean?

Relativity predicts different rates for the flow of time for observers who are in relative motion, but the omnipresent cosmic background radiation makes it theoretically possible to bring a spaceship anywhere in the Universe to absolute rest relative to this radiation, as any motion relative to it will make radiation coming from the direction of travel appear to have a higher frequency than radiation coming from the opposite direction. Would all observers in such spaceships share a common Now? Quantum mechanics also relies on a form of synchronised universal background time for all processes in the quantum phase, which precisely tracks the passage of time, along with all motion and rotation in space, yet is unobservable in our outer world. This mysterious feature of phase has been described as remarkable by many physicists, including Paul Dirac in his *Principles of Quantum Mechanics*, but it is something of an embarrassment to physicists — how do phase correlations between particles stay accurate, even when they are separated or in motion? It doesn't seem to fit with special relativity's changing clock speeds. All told, there is a great deal of confusion amongst physicists about this subject.

The following story is a lovely illustration of not only how a great physicist can be caught up in the prevailing mind-set, yet also be ready to admit his errors, and quickly update his thinking when presented with logical truths.

> The story concerns the so-called 'twin paradox' in relativity. In his book, Feynman had written "You can't make a spaceship clock, by any means whatsoever, that keeps time with the clocks at home." Now Fredkin happened to be teaching a course and this subject came up. In thinking about the paradox, Fredkin came up with a trivial way to make a spaceship clock that did keep time with the clock at home. Before making a fool of himself in front of his students, Fredkin thought he'd check with Feynman first. There was, of course, an ulterior motive for doing this and that was to 'sandbag' Feynman — a thing that Fredkin loved to do but rarely succeeded. The telephone conversation went something like this. Fredkin said "It says in your book that it is impossible for a clock on the spaceship to keep time with a clock at home. Is that correct?" Feynman replied "What it says in the book is absolutely correct." Having set him up, Fredkin countered "OK, but what if I made a clock this way ..." and then proceeded to describe how his proposed clock had knowledge of the whole trajectory and could be programmed to put the 'back

> home' time on the face of the clock. "Wouldn't that keep time with the clocks at home?" Feynman said "That is absolutely correct." Fredkin replied "Then what does that mean about what's in your book?" Feynman's instant response was "What it says in the book is absolutely wrong!"
> Anthony Hey, in his introduction to *Feynman and Computation*

Feynman's mental block arose because Einstein is considered to have abolished the idea of a Now, and it required Ed Fredkin's outsider's view and bravery to challenge Feynman and pierce his cultural bubble. Fredkin was largely self-taught and became a professor at MIT without having a university degree, he has also long espoused the idea that the Universe is a giant computer busy calculating something…

So, as Feynman admitted, just because the clock of observer A disagrees with the clock of observer B, this does not mean that they could not share a common pre-arranged moment without any communication between them. If you remain on Earth while your friend is travelling in a spaceship at 99% of the speed of light, her one second moment would last for seven seconds on your clock. The closer she got to light speed, the slower her clock would be running relative to yours. If you could ride on a photon of light, your clock would have stopped from the perspective of an observer — you would be travelling in the timeless Now.

Potentials and Gauges

Electromagnetism was described in Chapter 7 in terms of the electric and magnetic fields **E** and **H**, but there is an alternative formulation that uses two forms of potential: a *scalar potential V,* which is just a number without any directional information, and a *vector potential* **A**, which does have directional properties. The two formulations are equivalent, and physicists switch freely between the two, using whichever is most convenient for any particular calculation. However, there are rare situations where the potential formulation can account for effects that the field view seems to miss. A prime example is the Aharanov-Bohm effect, which describes a phase shift in the wave function of a charged particle as it passes close to a long solenoid, even though the magnetic field at the particle's location outside the solenoid is negligible. If there is no magnetic field there, how could it affect the particle's wave function? It turns out that in quantum mechanics, the potentials, unlike the fields, couple directly with wave functions, so the vector potential **A** *is able* to correctly describe the Aharanov-Bohm effect and the outcomes of experiments.

We will see in Chapter 15 how this direct coupling between wave function phase and electromagnetic potentials enabled Paul Dirac to infer the existence of magnetic monopoles.

An electric current flows through a conductor when there is a *potential difference*, or voltage, across it. When there is no magnetism present, the electric field **E** is equal to the rate of change of the scalar potential V in space: $\mathbf{E}$ = grad $V = \nabla V$, where ∇ is the gradient operator that picks out the direction in which the scalar potential V is changing most rapidly, as well as its rate of change. (The nabla symbol ∇ is also used as shorthand for div and curl: div $\mathbf{E} = \nabla \cdot \mathbf{E}$, curl $\mathbf{A} = \nabla \times \mathbf{A}$).

Stepping up from the representation of electromagnetism by the fields **E** and **H**, to the potentials V and **A**, brings a new freedom to choose between various *gauges*. (The word gauge has little significance). The magnetic field **H** is related to the vector potential **A** by the equation $\mathbf{H} = \nabla \times \mathbf{A}$**,** the magnetic field **H** being equal to the curl of the vector potential **A.** This grants the freedom to add any other vector to the vector potential **A**, without having any measurable effect at all on the magnetic field **H**, provided that the curl of the added vector is zero. (Note that potentials, unlike fields, are *not* measurable quantities). There is a rule of vector calculus that the curl of a gradient is always zero, so we can choose a scalar quantity λ, take its gradient, $\nabla\lambda$, and this will be guaranteed to have no curl, so we are free to add it to the vector potential **A**. Doing this will also have an effect on the scalar potential V, but will not effect the measurable electric field. Writing the transformed potentials as **A'** and V', the gauge transformation equations are written:

$$\mathbf{A}' = \mathbf{A} + \nabla\lambda$$

$$V' = V - \partial\lambda/\partial t$$

Electromagnetic Gauge Transformations

Physicists can switch between different gauges to suit particular problems, but the Coulomb gauge and Lorentz gauge are most commonly used. Following their origin in electromagnetic theory, gauge theories became very important in 20^{th} century physics. An interesting review of their origins can be found in Jackson and Okun's paper *Historical roots of gauge invariance*.

You may by now be wondering what this has to do with a universal NOW.

> There is a peculiar thing about the scalar potential in the Coulomb gauge: it is determined by the distribution of charge *right now*. If I move an electron in my laboratory, the potential V on the moon immediately records this change. That sounds particularly odd in the light of special relativity, which allows no

message to travel faster than the speed of light. The point is that V by itself is not a physically measurable quantity — all the man in the moon can measure is **E**, and that involves **A** as well. Somehow it is built into the vector potential, in the Coulomb gauge, that whereas V instantaneously reflects all changes in ρ, the combination $-\nabla V - (\partial \mathbf{A}/\partial t)$ does *not*; **E** will change only after sufficient time has elapsed for the "news" to arrive.
David J Griffiths, *Introduction to Electrodynamics*

Special Relativity or Lorentzian Relativity?

The book *Einstein, Relativity and Absolute Simultaneity* is a collection of essays by physicists and philosophers who find Einstein's special relativity, SR, lacking, and suggest that we should change to Lorentz's version of relativity, LR, which gives just the same answers to all calculations, but allows for a universal NOW, a preferred reference frame, and even the possibility of faster than light motion. This contrasts with SR, which allows no preferred reference frames — equality for all observers! — no aether, no NOW, and no faster than light travel. Here, we will mainly follow the chapter in the above book by Tom Van Flandern: *GPS & The Twins Paradox*.

Hendrik Lorentz published his Lorentz Ether Theory (LET) in 1904, a year before Einstein's SR, and it incorporated the relativity principle due to Henri Poincaré and earlier 19th century researchers, along with an aether, and the Lorentz transformations that also appear in SR, and are named after Lorentz. Lorentz's LET theory allowed the existence of an aether, and this was problematical as the Michelson-Morley experiment had failed to find the expected observable effects of the Earth's motion through an aether. But it was later realised that the local gravity field, acting as a preferred reference frame, could serve the same role as the aether. With this interpretation, but without any other changes to LET, Lorentz's theory is now called LR.

LR is much easier to understand than mind-boggling SR. For example, LR has no 'twins paradox' — a travelling astronaut ages more slowly than her twin who remains on Earth, simply because the stay-at-home twin keeps to the normal time set by the Earth's gravitational reference frame, while the flighty twin's passage of time is slowed because of her high speed motions relative to the Earth. LR does not claim that the reference frames of the two twins are equivalent, so there is no paradox. By contrast, in SR, because Einstein insisted on making all reference frames equivalent, the paradox arises because the flighty twin is equally entitled to say that the Earth twin has been moving relative to her, so the Earth twin should have aged more slowly because Earth time was slowed. SR can explain its way out of this, but only by complicated arguments that test credibility, as Van Flandern explains, requiring absurdities like the time and date changing in a distant place almost instantaneously. The most common 'explanation' of the paradox that is offered by devotees of SR, is that the travelling astronaut undergoes an acceleration to reach her outward speed, and an opposite acceleration to come back, and these destroy the reciprocity between the Earth and travelling frames. Van Flandern shows that this argument is unjustified, as experiments have found no effect on clocks, even with accelerations as high as 10^{19} times that of Earth's gravity at sea level. Accelerations do not change local clocks, clock rates, or biological aging.

The existence of a privileged reference frame in LR, which Einstein denied on principle, has proved to be crucial in designing the Global Positioning System, which relies on accurately synchronising the atomic clocks carried by orbiting GPS satellites to a single master clock. This high precision master clock is safely buried in a bunker in Colorado, and thus fixed to the Earth's gravitational reference frame. Einstein's prescribed method would require synchronising each moving clock to each other clock separately, a nigh on impossible task. Thanks to the equivalence of LR and SR, all the GPS satellites can be synchronised to their common, preferred reference frame — the Earth — and we can use GPS. But SR still dominates in physics despite the many dissenters, and experiments preferring LR over SR.

> Of critical importance to choosing the model that best represents nature, none of the eleven independent experiments testing SR verify frame reciprocity or distinguish SR from LR. In fact, historically, de Sitter, Sagnac, Michelson, and Ives concluded from their experiments that SR was falsified in favor of the Lorentz theory. …the GPS itself is a practical realisation of Lorentz's "universal time" wherein all clocks ("GPS clocks") remain synchronised despite being in many different frames with high relative speeds…
> How simple special relativity would have been all these years if physicists had realised that all they had to do was reset the clock rates so they all ticked at the same rate as the reference clock in the local gravity field!"
> Tom Van Flandern, *Einstein, Relativity and Absolute Simultaneity*

The Lorentz transformations that are used by both LR and SR, are implicit in Maxwell's equations of electro-magnetism which we saw in Chapter 7. Relativity is really a consequence of electro-magnetism, and before SR became dominant, both Lorentz and FitzGerald had interpreted the apparent shrinkage of a fast moving object along its line of motion as an electro-magnetic effect.

Van Flandern also discusses the possibility of faster than light communications which LR permits but SR forbids.

> Although LR has no intrinsic speed limit, it recognizes the innate difficulty of material bodies composed in part of electrons, while propagating in a luminiferous ether, being able to exceed the wave speed of that ether, the speed of light. LR treats this as analogous to a propeller-driven aircraft exceeding the speed of sound without any assists, such as from gravity. A force that cannot propagate itself faster than light cannot propel material bodies faster than light.
> Tom Van Flandern, *Einstein, Relativity and Absolute Simultaneity*

Many prominent physicists continued to support LR over SR, including Lorentz himself, and the very same Michelson, whose Michelson-Morley experiment was considered to have ruled out an aether and justified Einstein's SR. What seems to have won the day for SR was mainly Einstein's reputation, plus the Occam's razor simplicity of doing without a privileged reference frame, an aether, and a universal time. But razors can be dangerous, they can cut too much, and as Van Flandern shows, simplifying one thing can complicate matters in other ways.

The first postulate of Einstein's SR is the equivalence of all reference frames, which leads to the twins paradox as we saw above. The second postulate of SR is the fixed speed of light, which is not only independent of the speed of its source, like other waves, but also of the speed of the observer, unlike other waves. This last feature is unique to SR, but if the speed of light is set by the internal activity of electrons, this would not be surprising, as any observer must carry his own electron clocks around with him. In Chapter 13, we will look at the inner behaviour of electrons, the zitter dance, in which a fixed surface speed of light connects rotary internal motion with linear external motion. There seem to be two separate measures of time at work, bound together by the speed of light.

The Speed of Gravity

The sun up in the sky is not really where it appears to be — it takes about 8 minutes for the photons it emits to reach our eyeballs, and meanwhile the sun has moved on. We observe the sun at its previous position rather than its current position, because the information carried by light cannot propagate through space faster than the speed of light. Electro-magnetic forces depend on virtual photons to carry momentum between interacting bodies, and these are also restricted to the speed of light, so physicists have to allow for propagation delays by using retarded times and positions.

It is natural to think that the same speed limit should apply to gravity, especially as recent observations by LIGO confirm that gravitational waves also travel at light speed. But, strangely, a gravitational field appears to update itself at any distance instantaneously.

> The most amazing thing I was taught as a graduate student of celestial mechanics at Yale in the 1960s was that all gravitational interactions between bodies in all dynamical systems had to be taken as instantaneous. …as astronomers we were taught to calculate orbits using instantaneous forces; then extract the position of some body along its orbit at a time of interest, and calculate where that position would appear as seen from Earth by allowing for the finite propagation speed of light from there to here. It seemed incongruous to allow for the finite speed of light from the body to the Earth, but to take the effect of Earth's gravity on that same body as propagating from here to there instantaneously. Yet that was the procedure to get the right answers.
> Tom Van Flandern, *The Speed of Gravity — What the Experiments Say*

The instantaneous, non-local updating of a gravitational field is actually a necessity for orbits to remain stable.

> If the Sun attracts Jupiter towards its present position S, and Jupiter attracts the Sun towards its present position J, the two forces are in line and in balance. But if the Sun attracts Jupiter towards its previous position S′, and Jupiter attracts the Sun towards its previous position J′, when the force of attraction started out across the gulf, then the two forces give a couple [a turning force]. This couple will tend to increase the angular momentum of the system, and, acting cumulatively, will soon cause an appreciable change of period, disagreeing with observations if the speed is at all comparable with light.
> Arthur Eddington, *Space, Time and Gravitation*, quoted by Van Flandern

Eddington's argument is illustrated in Fig. 8.1, showing how attraction to the previous, retarded position of another orbiting body would create a nett turning force, or couple.

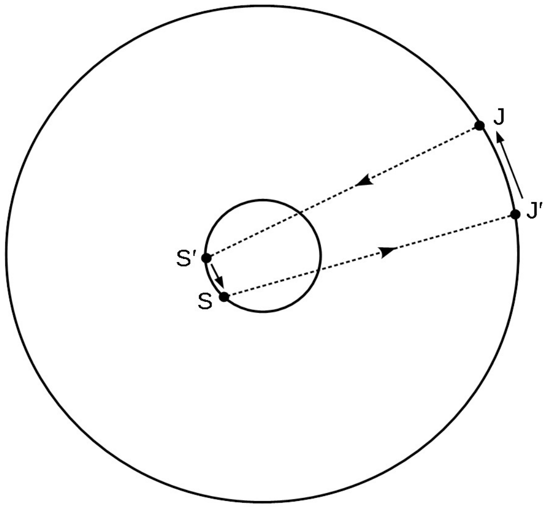


Fig. 8.1 Orbits are unstable with finite speed forces

There are several other effects that Van Flandern describes which require gravitational forces to be instantaneous, as they are in Newtonian gravity, or travel at more than 10^{10} times the speed of light in order not to disagree with experiments. Einstein's general relativity describes gravity as space-time curvature, so nothing needs to travel, and the inconvenient faster-than-light aspects are conveniently forgotten, as they are in the inflation theory of the early Universe, which requires space-time to expand much faster than light. A string can support tension along its length without anything moving, but tension in the string can also allow it to support wave motion when it is plucked, and this is a different effect. Van Flandern notes that there is no direct evidence that the gravity that keeps planets in orbit around the sun travels as gravitational waves. The gravitational waves that LIGO can now observe are extraordinarily weak, and it is possible that they are a residual electro-magnetic effect in disguise, only observable because they have an effect on the electrical particles that make up the atoms in detectors.

So, in summary, while there is evidence that gravitational waves travel at light speed, there is also evidence that gravitational influences travel at least 10^{10} times faster than light. It seems that two different forms of time are again at work, and one of them must operate within the Now.

Entanglement

The quantum mechanical phenomenon of entanglement must also operate within some form of universal Now. Entanglement was first highlighted by Einstein, Podolsky and Rosen in a 1935 paper that has become the most cited of any of Einstein's papers. Einstein's motivation was to show that quantum mechanics is incomplete, because it predicts the instantaneous 'spooky action at a distance' of entanglement, contravening the ban on any influence travelling faster than light that his own theory of special relativity insisted on.

A photon can divide into two separate photons that go off in different directions at the speed of light, but with entangled spins; whichever direction the spin of one is measured to choose, the other will be the opposite. Until one is measured, the spin of each is described as a superposition of the two possible states of quantum spin: spin up and spin down. The mystery of entanglement is that when one is measured and gives a random up or down result, the other which may now be far away, is always found to be in the opposite state — if the first photon is spin-up, the other will be spin-down, and vice versa. Experiments have shown that whatever influence passes between the first-measured photon and second-measured photon, it travels at least 30,000 times faster than light, and has in fact to be instantaneous, as paradoxical situations can otherwise occur. The two entangled photons seem in some way to be sharing in the Now of the first measurement. Quantum entanglement is a property of all quantum particles, not just photons, and has even been demonstrated for larger agglomerations of hundreds of atoms. There is more about spin and entanglement in Chapter 11.

Quantum mechanics also offers another faster-than-light effect in what is called the collapse of the wave function. Until quantum states are observed, they evolve in accordance with the Schrödinger equation, exploring all possibilities in parallel. But when a connection is made to the outer world in the form of a measurement, the wave function is said to 'collapse' instantly, breaking any superpositions, and creating a definite outcome. The collapse happens instantly, and across whatever distance in space that may separate the component particles of the wave function. Quantum mechanics has no answer as to how information can travel to make this possible.

Time and Eternity

Discussion of time, eternity, and Now, may appear to be just philosophy, but these ideas also come up naturally in Fourier analysis, which allows information to be represented in dual forms, and is one of the most important mathematical techniques in science and engineering — a smartphone relies as much on the Fourier Transforms performed by its DSP chip as it does on logic operations performed in its CPU. One type of Fourier analysis works by decomposing a signal into time domain information and frequency domain information, where frequency is just the reciprocal of time, 1/time. Each of the two domains contains *all* the information, but in very different forms that are useful for different purposes. For instance, it is simple to filter out unwanted frequencies in the frequency domain because each possible frequency has its own magnitude that can be reset to zero. In the time domain filtering is much harder because the frequency information is concealed.

Although Fourier analysis is one of the greatest treasures of mathematics, it is difficult to describe without complex mathematics. However, some familiarity with the idea of Fourier duality is essential to understanding the arguments of this book, as it shows just how a temporal order *can* co-exist with a non-temporal, how information that is *local* in one domain can be *global* in the dual domain, and how all this is central to quantum mechanics and the uncertainty principle, so I try to describe it in more detail in Chapters 10 and 11.

The eternal Now is essentially free of past and future.

> Past and future veil God from our sight;
> Burn up both of them with fire. How long
> Wilt thou be partitioned by these segments, like a reed?
> So long as the reed is partitioned, it is not privy to secrets,
> Nor is it vocal in response to lip and breathing.
> Jalal-uddin Rumi, quoted in *The Perennial* Philosophy

> If we take eternity to mean not infinite temporal duration but timelessness, then eternal life belongs to those who live in the present.
> Wittgenstein*, Tractatus Logico Philosophicus,* 6.4311

The modern world offers us ever more mental distractions that unfortunately do little to increase our happiness. In Vipassana (insight) meditation, it is possible to reach levels of concentration where the mind quietens, and thoughts that do arise can be effortlessly wafted away, along with concerns for past and future. Settling deeper into the eternal Now allows you to experience bliss that is unconditioned by anything other than sitting still and breathing. This can be as addictive as a drug, and the Buddhist monks and yogis who have mapped out these various levels warn of this danger. In Buddha's lifetime he had to face a mass protest of women who accused him of stealing their husbands. Buddha was very reluctant to allow women to become monks, and it is hardly surprising that women who were left to grow the food, care for the children, and sort out other worldly matters, resented that their menfolk did little else but sit around getting blissed out in meditation. This apparent lack of concern for worldly affairs in Buddhism is one of the reasons that it died out in India, becoming absorbed into Hinduism and surviving only in surrounding Asian countries. In the Hindu tradition it is common for someone to renounce the world and become a *sanyasin*, but only *after* their children are grown up.

The relentless pace of our modern world, and its disconnection from the Now and the eternal face of time, is one of the reasons we fear death. But what is there to fear if you have had a taste of the blissful peace that arises when no time passes, the *peace that passeth all understanding*? In *The Perennial Philosophy*, Huxley also blames our western obsession with time and progress for much of the violence perpetrated in the name of religions, Marxism, fascism and other philosophies. They all promise a worldly paradise in the future and use this to justify all manner of atrocities. Huxley points out that the eastern religions which regard time as cyclical, and are more grounded in the eternal Now, have historically been much less violent.

> ...most of the religions and philosophies that take time too seriously are correlated with political theories that inculcate and justify the use of large-scale violence.
> Aldous Huxley, *The Perennial Philosophy*

St. Augustine struggled to understand time and found it an impossible task, but he recognised that the Now is a form of eternity, and expressed the hope that people would be able to disconnect from their discursive thinking, and experience for themselves the power of eternity in the Now.

> Try as they may to savour the taste of eternity, their thoughts still twist and turn upon the ebb and flow of things in past and future time. But if only their minds could be seized and held steady, they would be still for a while and, for that short moment, they would glimpse the splendour of eternity which is for ever still. They would contrast it with time, which is never still, and see that it is not comparable. They would see that time derives its length only from a

great number of movements constantly following one another into the past, because they cannot all continue at once. But in eternity, nothing moves into the past: all is present. Time, on the other hand, is never all present at once. The past is always driven on by the future, the future always follows on the heels of the past, and both the past and the future have their beginning and end in the eternal present. If only men's minds could be seized and made still! They would see how eternity, in which there is neither past nor future, determines both past and future time. Could mine be the hand strong enough to seize the minds of men? Could any words of mine have power to achieve so great a task?

St. Augustine, *Confessions*

9
Computation

Landauer’s Principle

> Information is inevitably tied to a physical representation. It can be engraved on stone tablets, denoted by spin up or down, a charge present or absent, a hole punched in a card, or many other alternative phenomena. It is not just an abstract entity; it does not exist except through a physical embodiment. It is, therefore, tied to the laws of physics and the parts available to us in our real physical Universe.
> Rolf Landauer, *Information is Inevitably Physical*, in *Feynman and Computation*

Landauer’s principle that you cannot have information without it being represented in a physical manifestation can be read the other way: any physical entity is a manifestation of information.

Computers are Simple

It may seem strange to describe something as complex as a computer as simple, but the basic building blocks required to make a computer are surprisingly simple. In the mid 19th Century, Charles Babbage designed a mechanical computer, and tried to build it, but with the technology of the time it was not possible to machine the large numbers of parts required with sufficient precision, so it was never completed. It was only the advent of electronics a century later that launched the computer age. The essential things that flow through a computer are bits, binary digits which can each take one of two values, 1 or 0, represented in electronic computers by electric charges and currents. The two values of a bit can also represent TRUE or FALSE, YES or NO, or any other pair of opposites you choose. We currently use a few thousand electrons to represent each bit, but the number required is dropping each year as we work down towards the atomic level. Of course, nature has always worked at the level of atoms and individual sub-atomic particles, and that is part of the extraordinary power of natural computing! There are many types of computer including: Turing Machines, Finite State Machines, Cellular Automata, Reversible Computers and Quantum Computers.

Turing Machines

In the 1930s, Alan Turing thought up his Turing machine, and showed that a particular type called a Universal Turing Machine, could mimic any other Turing machine, and so perform any computational task — it is a *universal* computer. It turns out to be quite simple to design a universal computer. We will look at various types, but apart from quantum computers, they are all equivalent to a Universal Turing Machine. Although the Turing machine is a theoretical model, it provides a good illustration of the simplicity of computation.

The Turing machine of Fig. 9.1 consists of a paper tape, of unlimited length, which can have symbols written on it. It is only universal if the tape is potentially infinite in length, but only a few symbols are required. The tape can move one step at a time in either direction and line up the next symbol after a read-write operation of the head. The head reads one symbol from the tape, and another symbol showing the current choice from the finite number of internal states. A look-up table tells the machine what to do for each pair of symbols. The head writes a new symbol on the tape, which then moves one step left or right, and the internal state of the machine is also updated to one of its finite list of possibilities. The machine will finally print the results of its computation somewhere on the tape and stop. The symbols used in the diagram have no significance, they are purely for illustration.

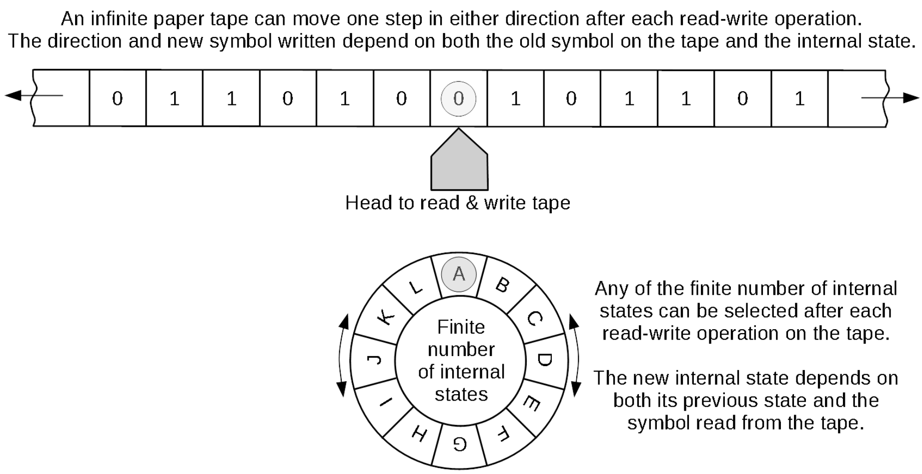
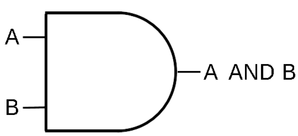


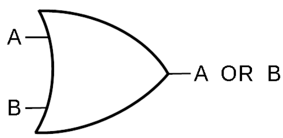
Fig. 9.1 A Turing Machine
An infinite paper tape of symbols interacts with a finite number of symbols.

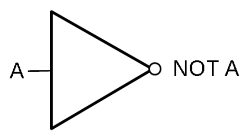
But sometimes the machine might never stop, because the problem is uncomputable, and Turing showed that it is sometimes impossible to prove that his machine will halt. This is known as the halting problem, and Turing had been influenced by Kurt Gödel, whose 1931 incompleteness theorems showed that there are more truths than can be proved from the rules of arithmetic, or any other fixed set of rules. The simple interaction of a finite set of states, with an infinite set of states in a Turing machine provides an example of the infinity of the kingdom of truth.

AND, OR and NOT

Even our most powerful computers do no more, essentially, than push large number of bits through large numbers of standard logic gates. There are many types of logic gate, but only three types need to be wired together to build an arbitrarily powerful 'universal' computer: AND, OR and NOT. These gates are the holy trinity of what is now known as Boolean logic. A NOT gate has one input and one output and changes a 0 to 1 and vice versa. An AND gate has two or more inputs and gives 0 as its output unless all the inputs are 1, in which case it outputs 1. An OR gate has two or more inputs, and outputs 0 unless at least one of the inputs is 1, in which case it outputs 1. Here are the symbols for these gates and their 'truth tables' which tabulate what they output for any given input:







AND Gate

Inputs		Output
A	B	A AND B
0	0	0
0	1	0
1	0	0
1	1	1

OR Gate

Inputs		Output
A	B	A OR B
0	0	0
0	1	1
1	0	1
1	1	1

NOT Gate

Input	Output
A	NOT A
0	1
1	0

Once you have a supply of these basic logic gates you only need to connect them together in the right way to create a computer as powerful as you wish. Some chips are made by stacking layers of basic gates, with each layer consisting of large numbers of a single type of gate such as AND. Any logic circuit can then be formed by interconnecting the layers of the chip in the appropriate way.

Clocking

Almost all computers rely on pulses from a master clock to keep every process synchronized. A crystal oscillator is used to provide a fundamental frequency, and this is divided down to provide the timing of a master clock.

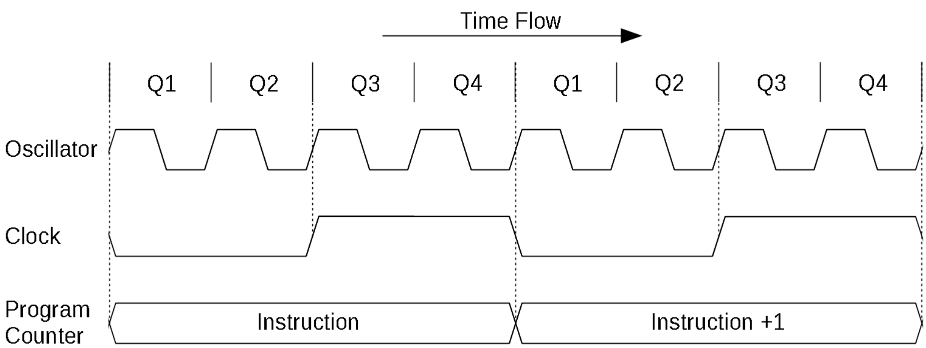


Fig 9.2 A microprocessor timing diagram

In the simplified microprocessor timing diagram of fig. 9.2, the clock that times the processing of instructions runs at a quarter of the master oscillator frequency, so a clock cycle that processes one instruction takes four cycles of the oscillator. Engineers use timing diagrams like this to ensure that processes do not interfere with each other. For example, you cannot write to a memory and read from it at the same time, there needs to be a certain separation between these processes, and enough time for values to settle.

Electronic circuits are very fast, but it still takes time for current to flow into or out of a logic gate, and the gate must have stable inputs before you ask it to process the inputs, otherwise its behaviour would be unpredictable. The sloping lines on the edges of the pulses in Fig. 9.2 give an indication of the finite time it takes for a transition to take place. The signal from the central clock also takes different amounts of time to reach the various gates in a circuit, so allowance must be made for this and other factors that could prevent stable operation. One solution is to use a complete cycle of the master clock to instruct every gate to take a single step in its own four phase cycle that we can describe as follows:

1) Read the inputs

2) Process the inputs

3) Write to the outputs

4) Clear the inputs ready for the next cycle

Making all the gates march in step like this ensures that none of the gates try to read an input from another gate whilst that one is writing its output, and also allows time for each input and output to stabilise before being read. It is possible, but much more difficult, to build asynchronous logic circuits that do not need a central clock.

In Chapter 4 we saw how the complex exponential also steps endlessly around the 4 phase cycle of Real, Imaginary, Negative Real, and Negative Imaginary, and that the logical operation cycles that the Margolus-Levitin theorem counts are cycles within every particle around these inner four compartments of the unit 1.

Serial and Parallel Computing

Most computers that we use today are still based on the original architecture that John von Neumann devised in 1945. A central processing unit (CPU) sucks in an instruction and some data, acts on the data according to the instruction, then sends the data out again. It is like an empire run by a single emperor who has to do everything himself. This is called serial computing — one thing at a time. The whole thing is fast only because electronic circuits can run at billions of operations per second. There have been many attempts to build parallel computers in which many processors work on parts of a problem at the same time, and there are numerous different types. The commonest type of parallel processing chips, called GPUs, are designed for graphics processing, and speed up the processing of pictures by applying the same instruction to multiple pixels simultaneously. CPUs containing multiple processor cores are now common, but need to be controlled very carefully to keep the overall computing task under control. It is just the same as with humans — it is often easier to do a task yourself than to organise a group of people to do it.

Cellular Automata (CAs)

The simplest type of parallel computer is the Cellular Automaton, also a brainchild of John von Neumann, following suggestions from Stanislaw Ulam. Some physicists, like Ed Fredkin, believe that the Universe is a giant CA. In each time step, every cell in a grid is updated together, in parallel, according to what states its neighbours are in. A CA is easy to simulate on a standard computer. Here is a simple example of a CA with a grid of cells in just one dimension, each having possible states 0 or 1.

1	0	1	0	1	1	1	0	0	1

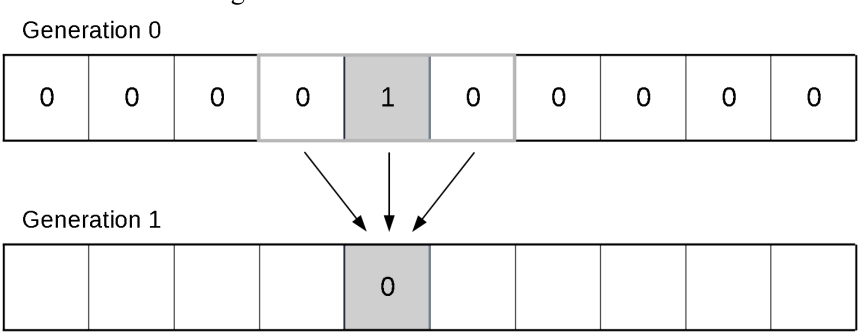
For each cell of this CA, we can define a neighbourhood around it, which is picked out below in pale grey. In this example, the neighbourhood is just the cell itself, shaded grey, and one cell on each side. (Assume that the end cells have a neighbouring cell of 0 on their outside).

1	0	1	0	1	1	1	0	0	1

For each step in time there is a common set of rules that tells each cell how to update itself. The three bits in our neighbourhood here, provide the eight possible choices shown by this rule set:

000	001	010	011	100	101	110	111
↓	↓	↓	↓	↓	↓	↓	↓
0	1	0	1	1	0	1	0

Operation: We start the CA off at generation 0, with a 1 near the middle, and apply the rules for each cell and its neighbours to get the new value for that cell. The particular cell highlighted below and its neighbours start off with 010. According to the rule for 010 in our rule set, this means that the highlighted cell must change from 1 to 0 for the next generation.

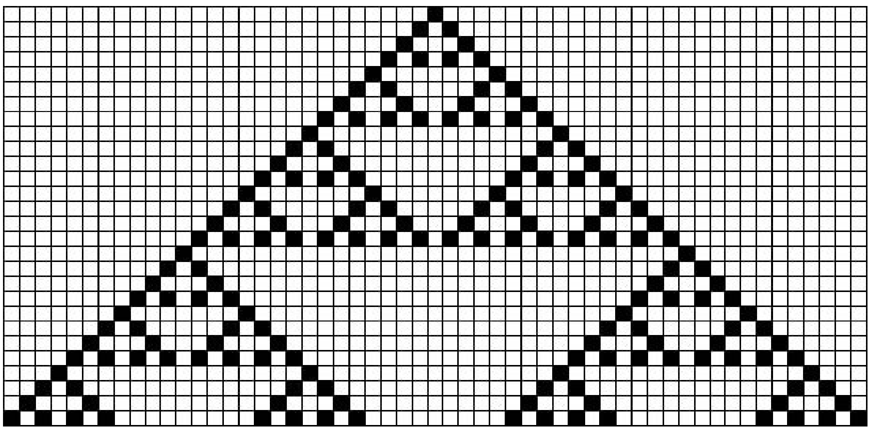


Applying this procedure to each cell in generation 0, we get the following pattern for generation 1 cells:

Generation 1

0	0	0	1	0	1	0	0	0	0

We can then simply repeat the cycle again and again, creating a new row of cells for each generation, rather as a carpet emerges from a loom. Although this is a one dimensional CA, the emerging pattern is two dimensional. If you use a larger 'canvas' of cells, and keep repeating the above procedure, this fractal pattern emerges:



Smaller pixels, and much more processing, give the Sierpinski triangle of Fig 9.3.

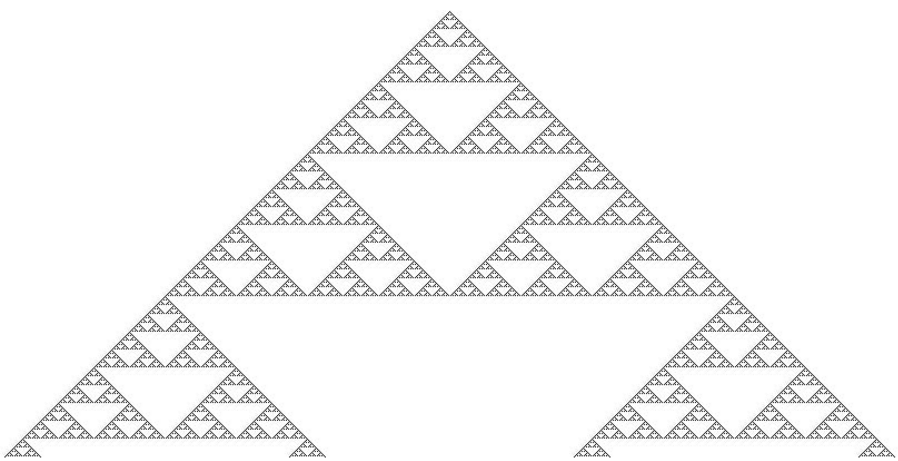


Fig. 9.3 The Sierpinski triangle

In his book *A New Kind of Science*, Stephen Wolfram explored the different patterns that a simple CA can produce with different rule sets and different starting patterns. His rule 90, used above, produces the fractal Sierpinski Triangle, self-similar at different scales. Surprisingly, with the same single black starting pixel, and just a small change to the rule set (Wolfram's rule 30), the pattern produced is irregular and complex and continues to be so even after a million steps. Some simple starting conditions and many repetitions of a simple process can lead to infinite complexity.

Cellular Automata make useful tools for simulating all sorts of physical processes. To simulate processes in normal space, you need a CA working in three dimensions, rather than the one dimensional CA above.

> From the point of view of a physicist, a CA model is a fully discrete classical field theory. Space is discrete, time is discrete, and the state at each lattice point has only a finite number of possible discrete values. The most essential property of CA's is that they emulate the spatial locality of physical law: The state at a given lattice site depends only upon the previous state at nearby neighboring sites. You can think of a CA computation as a regular space-time crystal of processing events: A regular pattern of communication and logic events that is repeated in space and in time. Of course it is only the structure of the computer that is regular, not the patterns of data that evolve within it! These patterns can become arbitrarily complicated.
>
> . . . in the distant future I expect that our most powerful large-scale general purpose computers will be built out of macroscopic crystalline arrays of identical invertible computing elements.
>
> Norman Margolus, *Crystalline Computation*, in *Feynman and Computation*

Reversible Computers

The NOT gate shown above has one input and one output, one bit goes in and one comes out, no information is lost, and the NOT process is reversible. But AND and OR gates have at least two inputs and only one output, so more bits go in than come out, and the process is irreversible. All our current day computers are irreversible, and the discarded bits account for some of the heat that processor chips dissipate. The truth table for the OR gate above shows that if you have an output of 1, there are three different input combinations that give the same result: 0 1, 1 0 and 1 1, so it is not possible to progress logically in reverse, from output to inputs.

> The great discovery of Bennett and, independently, of Fredkin, is that it is possible to do computation with a different kind of fundamental gate unit, namely, a reversible gate unit.
>
> Richard Feynman, *Computing Machines in the Future*, in *Feynman and Computation*

> Ed Fredkin pursued the idea that information must be finite in density. One day he announced that things must be even more simple than that. He said that he was going to assume that information itself is conserved. "You're out of your mind, Ed," I pronounced. "That's completely ridiculous. Nothing could happen in such a world. There couldn't even be logical gates. No decisions could ever be made." But when Fredkin gets one of his ideas, he's immune to objections like that; indeed they fuel him with energy. Soon he went on to assume that information processing must also be reversible — and invented what's now known as the Fredkin gate.
>
> When Bennett and Landauer wrote their paper showing that reversible, non-dissipative computation was indeed possible in principle, I was asked to be a referee for the journal they submitted it to. I read the paper over and over — every day for a solid month. Finally I sent a note to the journal. "This result just doesn't seem possible. However, I have read it very carefully, and cannot find where might be the mistake. I suppose you'll just have to publish it!"
>
> Feynman too was skeptical. However, instead of merely checking their proof, he set out to prove it for himself. He came up with a different and simpler proof, which convinced many others that the discovery was believable. Soon he started to design the first quantum computers.
>
> Marvin Minsky, *Feynman and the Cellular Vacuum*, in *Feynman and Computation*

There is more than one type of reversible gate, but I personally find the Fredkin gate of Fig. 9.4 quite awesome in its simplicity and power. Not only is it reversible, but it is also complete, in that by just putting together Fredkin gates, you can build an arbitrarily powerful, universal, reversible computer.

The input on line A passes unchanged to the output A'. The inputs on B and C are output to B' and C', the state of A determining whether B' and C' are swapped. If A = 0, the state that goes in on B comes out on B', and the value that goes in on C comes out on C'. If A = 1, then the state that goes in on B comes out on C', and the state that goes in on C comes out on B'.

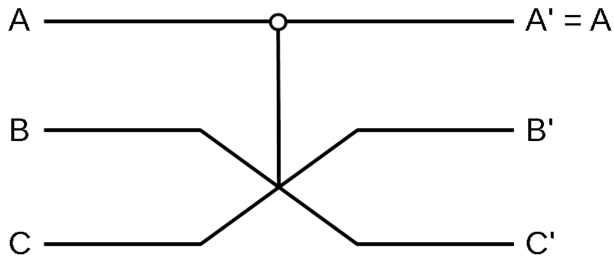


Fig. 9.4 A Fredkin gate. A controlled exchange.

> Fredkin added an extra constraint on the outputs and inputs of the gates he considered. He demanded that not only must a gate be reversible, but the number of 1s and 0s should never change. There is no good reason for doing this, but he did it anyway. He introduced a gate performing a controlled exchange operation.
>
> Richard Feynman, *Lectures on Computation*

Information is also conserved in the phase space that is used by analytical mechanics and quantum mechanics as a consequence of following the principle of least action. Nature's information is fully recyclable, and this was a prime motivation for Fredkin in inventing his gate — but he faced opposition.

> In deeming the Fredkin gate nonsense, the faculty members [at MIT] had their choice of rationales. The first was guilt by association: Ed Fredkin has this quasi-religious conviction that the universe is a computer, and the Fredkin gate is somehow tied in to this whole thing. The second was that Fredkin almost never publishes his ideas.
>
> … There was another thing that made it easy to ignore the reversible computer: Fredkin still hadn't come up with a clear *reversible* computer. It is one thing to describe a Fredkin gate and then argue abstractly that a suitable arrangement of such gates could do anything any other computer could do. It is another thing to actually design a nice, simple computer that clearly could work and clearly would not discard information. This point was made with particular force by a doubter named Paul Penfield, a professor of electrical engineering at MIT. "This guy in some sense got on my case", recalls Fredkin, "So, I got mad and I decided I was going to find a simple physical realization."
>
> Robert Wright, *Three Scientists and their Gods*

What to do with the Garbage?

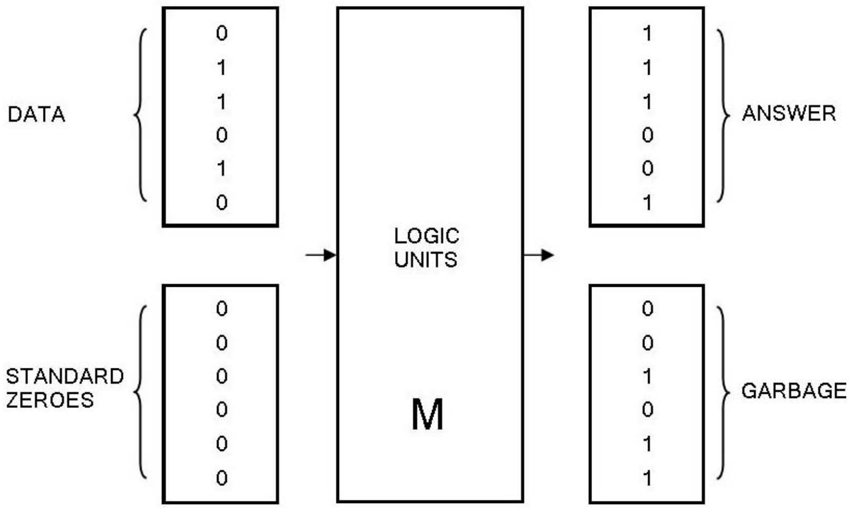


Fig. 9.5 Feynman's general reversible computer.

In his *Lectures on Computation*, Feynman represents a general reversible computer in the form of a diagram like Fig. 9.5. The inputs to the logic units are the data to be processed and a pile of standard zeros. The output is the answer, consisting of the same number of bits as the data going in, plus a pile of garbage bits. But the 'garbage' actually contains useful information if you want to run the logic backwards. This is something you cannot do with a computer built from conventional gates like AND and OR which take in two or more bits, but only output one bit — information is always lost. With a reversible machine, you can use the answer plus the garbage to work backwards to the inputs.

The irreversible computers that we use today are restricted by their ability to get rid of the heat they create, and some of this heat energy comes from discarded information. A kind of wall has been hit in recent years in terms of the clock speeds of processors, because of the problem of heat production. The faster you clock the device, the more heat it needs to get rid of, so the recent emphasis has been on adding more processor cores rather than trying to make a single core run faster.

The logical operations within particles that are counted by the Margolus-Levitin theorem are fully reversible — rest-mass energy does not leak out of atoms! It was the discovery that reversible computation is possible that opened the way to understanding the internal energy of particles as computation.

The Billiard Ball Computer

The Billiard Ball Computer is not a practical machine, but a type of reversible computer that was invented by Fredkin, Toffoli and others as an idealised model to study. The movement of billiard balls on a frictionless plane is used to represent the movement of bits through logic gates. Billiard balls are fired into the machine to represent the input, and the distribution of balls coming out gives the output. The presence or absence of a ball at a particular location and time corresponds to 1 or 0.

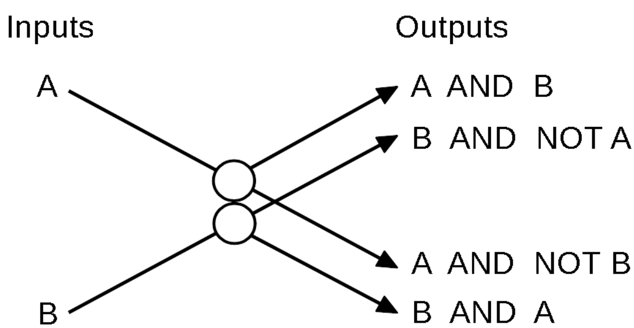


Fig. 9.6 Basic two ball collision computation.

You can see how the logic works in Fig. 9.6 if you understand that the presence of a ball from input A at a crossing makes the A input 1, the absence of a ball from A at the crossing makes the A input 0. The same rules apply for a ball from input B. So if, for example, both inputs have a ball entering, the inputs are 1 and 1, the balls must collide, and one ball goes to each of the outputs labelled A AND B, and B AND A. If a ball goes in on A, but no ball goes in on B, the A ball sails straight through without any collision and comes out at the output A AND NOT B.

It is hard to look at this model without being reminded of the ceaseless collisions of gas molecules, and how the pioneers of thermodynamics had to include entropy in their equations. Entropy just measures the amount of information about the position and momentum of each molecule that is hidden from our macroscopic viewpoint.

Neural Networks & Artificial Intelligence

Another variety of computation is performed by the neural networks that are at the heart of machines designed for Artificial Intelligence applications. There are many different types of neural network, including the type installed in every human skull. To get some idea of how a neural network operates, we will look at a very simple type that can recognise handwritten digits from 0 to 9.

The example shown in fig. 9.7 is adapted from the video of 3Blue1Brown here:

https://youtu.be/aircAruvnKk

which describes the workings of this neural network using beautiful, animated graphics and simple language, and also includes the mathematics for those who are interested.

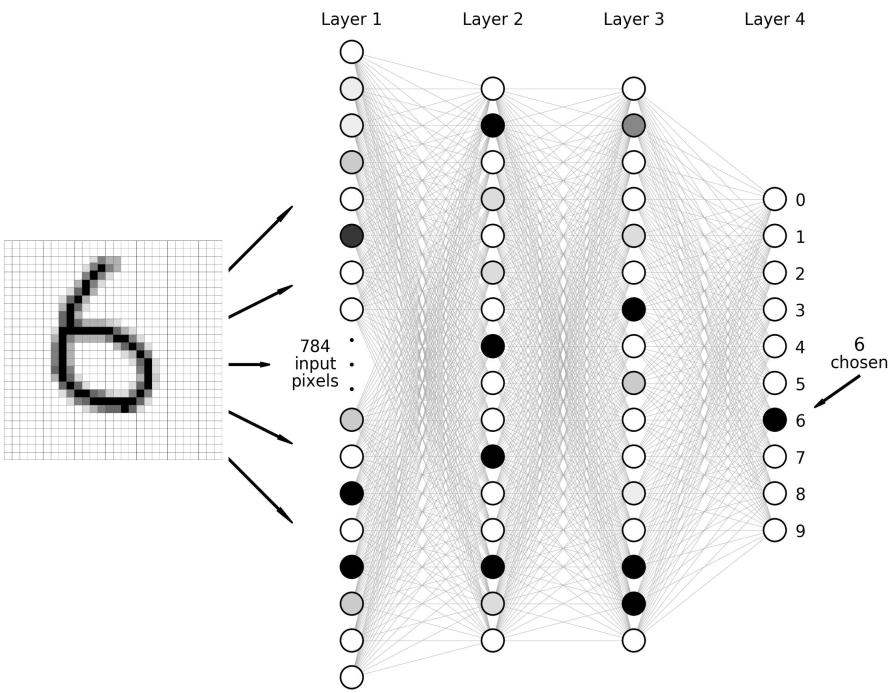


Fig. 9.7 A neural network to recognise the numbers 0-10.

The neural network of Fig. 9.7 consists of four layers which are shown as four vertical columns of 'neurons'. Activation levels of neurons are shown as shades of grey. We will assume that the network has already been trained by showing it a large number of pictures of handwritten numbers that have been labelled by humans as to their content. The training sets the optimum strengths for all the connections between neurons. Note that the thin lines showing these connections between the neurons in the diagram are all drawn the same, and do not show the varying strengths of these connections. We will not go into the details of training, but the technique of *back propagation* that is used is explained in later videos of the

3Blue1Brown series in the link above. In the current example of number recognition, information feeds from left to right. In back propagation training, information feeds from right to left.

Our sample 28 x 28 pixel picture of a hand-drawn 6 is shown on the left. The fourth column of 10 neurons on the right in layer 4, are the output choices from 0 to 9 that the network must pick from to identify the number in the input picture. The first column, on the left, is the input layer 1, which has 784 neurons corresponding to the 28 x 28 = 784 pixels of the picture. The activation levels of each neuron in this first layer represent how dark that pixel is on a grey-scale between white and black. The two intermediate layers 2 and 3 of neurons in the middle columns of the network, are shown here with 16 neurons each, but the actual number of neurons in these layers is not important.

Activation levels of the 784 first layer neurons propagate to the right, along the connections to the second layer of neurons, where each neuron's activation level is set by the *combined* effect of all its inputs. The second layer neuron activation levels then propagate to the right again, and influence the activation levels of the third layer neurons. This process repeats once more, the third layer neuron activation levels propagate to the right, and will set just one neuron in the fourth layer, providing that the number in the input picture is recognisable, and the network has had sufficient training. The hand-drawn 6 is recognised.

You may wonder what the two middle layers do here. This is very difficult to understand, and for more complex networks, what the neurons of intermediate layers 'see' is impossible to understand. But the fact is that it works. This simple network can 'understand' handwritten digits as well as a human. Similar methods are used in advanced A.I. applications that can recognise faces in security camera pictures.

A pair of A.I. systems can learn by a process that uses a form of Darwinian evolution. This is how, in 2017, the AlphaZero A.I. taught itself in just a few hours to play chess better than any human, after starting with no information apart from the bare rules of the game. A pair of neural networks competed against each other through millions of games of chess, with the winner of each game keeping its connection strengths for the next game, whilst the loser used these same winning connection strengths, but with small random changes.

It is hard to understand how people can deny the power of Darwinian evolution in the face of this evidence. Evolution is powered by the interplay between an organism and its background ecosystem, each steering the other. A.I.s like AlphaZero use a pair of neural networks. This is also how the Universe itself learns, by holding all its information in dual forms that evolve together, rather like AlphaZero.

Quantum Computers

The next great leap for computation after reversibility, is from conventional bits, to the qubits of quantum computers. The idea of quantum computers was first suggested by Feynman, but he was more concerned with the question of what sort of computer would be needed to simulate the subtleties of quantum mechanics. He concluded that conventional computers, however powerful, are not up to the job, and suggested that we would need to use natural quantum processes to simulate other quantum processes.

The main problem for conventional computers is that they use bits. Each individual bit in a computer register can take the value of 0 or 1. But the register as a whole can only have a single value, So an eight bit register can only have one of 256 (2^8) possible values, a sixteen bit register can only have one of 65536 (2^{16}) possible values, etc.

The great advantage of quantum computers is that they use qubits which can be not just 0 or 1, but both of these, and all shades in between, at the same time. So a qubit register of eight bits can represent all 256 possibilities simultaneously. As you might expect this gives a quantum computer a considerable advantage, but you still have to get around the problem that any measurement of these subtle quantum states collapses them, so that you only get a single bit output of 0 or 1.

When Peter Shor published his quantum factorisation algorithm in 1994, showing how a quantum computer could factorise a number with unprecedented speed, and produce a real output, the race to build quantum computers got into full swing. The central problem in building a quantum computer is to isolate whatever is holding your qubits from the surroundings. Any contact with the outer world can cause decoherence, and destroy the subtle interference effects that quantum computation requires.

> Among the many technical difficulties of working at the level of a single atom or single electron, one of the most important is that of preventing the environment from being affected by the different interfering sub-computations. For if a group of atoms is undergoing an interference phenomenon, and they differentially affect other atoms in the environment, then the interference can no longer be detected by measurements of the original group alone, and the group is no longer performing any useful quantum computation. This is called

> decoherence. I must add that this problem is often presented the wrong way round: we are told that 'quantum interference is a very delicate process, and must be shielded from all outside influences'. This is wrong. Outside influences could cause minor imperfections, but it is the effect of the quantum computation on the outside world that causes decoherence.
> David Deutsch, *The Fabric of Reality*

Deutsch is saying that the quantum superpositions are very stable, and not easily contaminated by information entering from outside, but if the superposed quantum information leaks out into the surroundings, you would need to include those surroundings in any attempt to recover the superposed information.

The Square Root of NOT

We have already looked at the standard logic gates of AND, OR and NOT, but quantum computation adds a new type of gate that does not exist according to normal logic, the square root of NOT, or √NOT. It has a 50% chance of performing a NOT and flipping the bit, so it behaves like a switch that randomly chooses an outcome of 0 or 1. This is illustrated in Fig. 9.8.

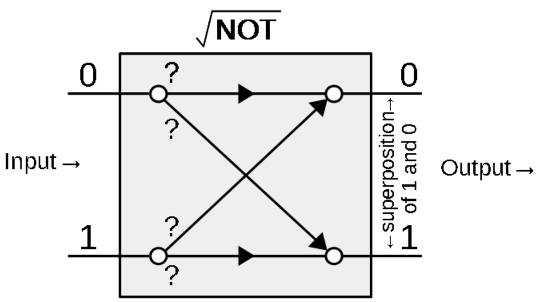


Fig. 9.8 A √NOT gate gives random outputs: 50% 0, 50% 1.

The strange fact is that if you join two of these gates together, doing the √NOT operation twice in succession, you do get a definite pure NOT of the original input, there is no randomness in the value of the output. This defies ordinary logic in which two successive random events cannot result in a definite outcome. Fig. 9.9 illustrates these results.

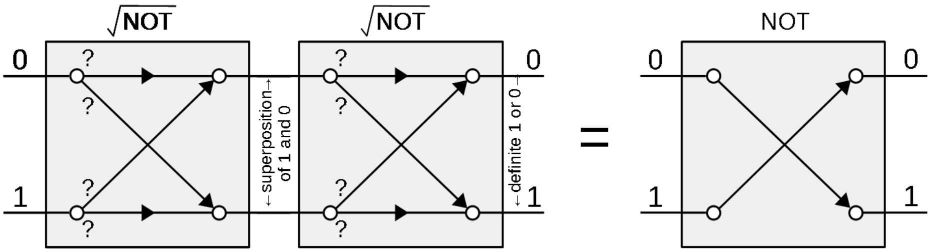
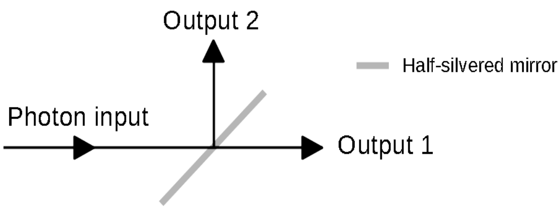


Fig. 9.9 Two successive √NOT gates give a definite 1 or 0 output.

A √NOT gate may sound very abstract, but the √NOT gate turns out to have a simple physical realisation in nature — a half-silvered mirror that reflects half the light that impinges on it, and transmits the other half. This is illustrated in Fig. 9.10.

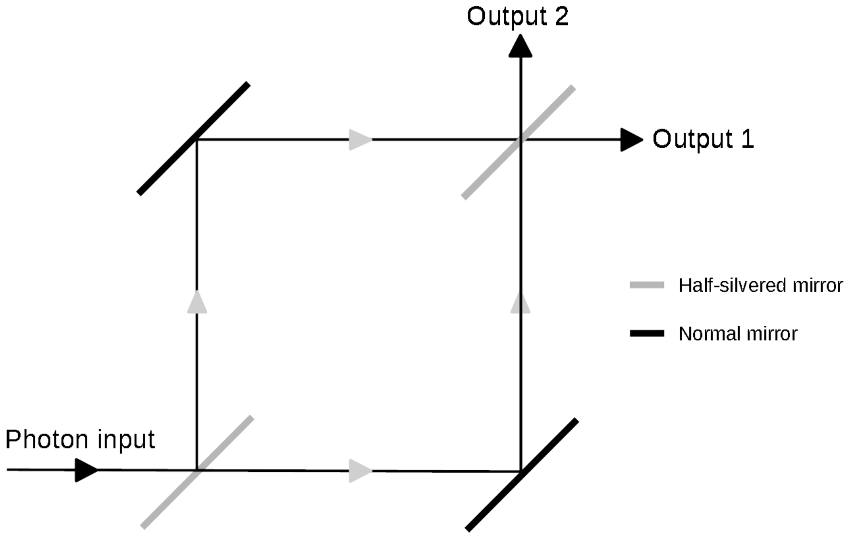


This is a physical representation of a √NOT gate.
A half-silvered mirror reflects half the light and transmits half of the light impinging on it.
But a photon cannot split and is always detected either at Output 1 or Output 2.

Fig. 9.10 A half-silvered mirror acts as a √NOT gate.

Each individual photon must actually take both routes. This is demonstrated by the interferometer arrangement shown in Fig. 9.11, which passes a photon through two half-silvered mirrors, i.e. two √NOT gates in succession.

A photon has a quantum mechanical 'amplitude' to be following either path through the interferometer. The amplitude is related to the probability of the photon actually being detected at any point, but whereas probability values are always between 1 for always, and 0 for never, quantum amplitudes are described by the two compartment, complex numbers that we looked at in Chapter 4. These amplitudes have real and imaginary parts, and can also be positive or negative, this is what allows wavelike interference effects.



A physical representation of two concatenated √NOT gates.
If a photon took one path or the other through the interferometer, it should arrive half the time at Output 1 and half at Output 2. But experiments show that it always arrives at Output 1. This shows that a single photon must take both paths!

Fig. 9.11 A photon passing through two concatenated √NOT gates

The cancellation that prevents a photon ever reaching Output 2 in Fig. 9.11 comes from adding a positive amplitude from one path, to a negative amplitude from the other. Adding, or superposing, complex amplitudes is the normal procedure in quantum mechanics up until something in the outer world observes or measures the quantum system. On measurement, the real world outcome probability is found by summing the superposed amplitudes and squaring the result. As we saw in Chapter 4, the process of squaring a complex amplitude has the effect of getting rid of the imaginary and negative terms in the amplitude and giving a real, positive probability.

10
Fourier Duality

A Dual Perspective

Joseph Fourier's publication of *The Analytical Theory of Heat* in 1822 is considered one of the great milestones in the history of mathematics. James Clark Maxwell called Fourier's book "a great mathematical poem". While Fourier analysis is used throughout physics, engineering and many other fields, most people have never heard of it, or the extraordinary dual nature of information that it reveals. This is a pity, as I believe it gives more insight into the nature of time than all the philosophers who have struggled with this subject. Time and its reciprocal, frequency, are bound together in a dual relationship, as are space and inverse space, and we need some understanding of how these dualities operate if we want a better understanding of the world. This is not an easy subject, but I try here to give some insight into it with as little mathematics as possible.

> With the discovery of quantum mechanics, it became clear that Fourier analysis is the language of nature itself. On the "physical space" side of the Fourier transform, one can talk about an elementary particle's position; on the other side, in "Fourier space," one can talk about its momentum or think of it as a wave. The modern realisation that matter at very small scales behaves differently from matter on a human scale — that an elementary particle does not simultaneously have a precise position and a precise momentum — is a natural consequence of Fourier analysis.
> Barbara Burke Hubbard, *The World according to Wavelets*

Most texts on Fourier analysis are completely inaccessible to non-mathematicians, but Barbara Hubbard's book, *The World According to Wavelets*, is a rare exception, providing deep insights into Fourier duality for all levels of readers. The wavelets of her title are the little "wave packets" that have become popular in recent years for probing data in both time and frequency at once. Elementary particles are described in quantum mechanics by similar wave packets, which also represent a particle's position and momentum as Fourier transforms of one another. The famous Heisenberg Uncertainty Principle is not just a mysterious peculiarity of quantum mechanics as we have been led to believe, but shows up in much simpler everyday things like sound, or signals, or anywhere else where Fourier duality comes into play. For example, the more accurately you want to measure the time that a musical note is played, the less accurately you can know its pitch, the uncertainty principle applies here too.

For a hundred years before Fourier, Newton's differential equations had allowed many physical processes to be described mathematically, but many of the resulting equations remained unsolvable, including the equation that describes heat flow. Fourier analysis allows such an equation to be recast in an alternative form, which is sometimes easily solvable. First you transform the data into the dual Fourier space where the equations can be solved, then you transform it back to our familiar space. While Fourier originally applied his method to problems of heat flow, his technique is so versatile and powerful that it is now used throughout science and engineering, wherever there is data that needs to be analysed, filtered, or compressed…, the list of applications is endless. Transforming data between the dual Fourier domains takes a lot of number crunching, and that is why every smartphone has a Digital Signal Processing (DSP) chip, specialising in this task.

> The Fourier transform is the mathematical procedure that breaks up a function into the frequencies that compose it, as a prism breaks up light into colors. It transforms a function that depends on time into a new function which depends on frequency. This new function is called the Fourier transform of the original function.
>
> . . . A function and its Fourier Transform are two faces of the same information. The function displays the time information and hides the information about frequencies. The function corresponding to a musical recording shows how the air pressure (produced by sound waves) changes with time, but it doesn't tell us what frequencies — what notes — make up the music. The Fourier transform displays information about frequencies and hides the time information: the Fourier transform of the music tells us what notes are played, but it is extremely difficult to figure out when they are played.
>
> Nevertheless, the function and its transform both contain all the information of the signal. We can compute the transform from the function and then retrace our steps, reconstructing the function from the transform.
> Barbara Burke Hubbard, *The World according to Wavelets*

Going Digital

The digital revolution started in 1900 when Planck reluctantly introduced the quantum of energy, to account for the spectrum of heat radiation. To get an equation that agreed with experiments, he had to describe the energy as being emitted in chunks whose size depended on the frequency of the radiation. For many years, Einstein ran with this quantisation idea more than anyone else, and it led to some of his greatest breakthroughs. From his work on Brownian motion, the microscopic jiggling motion of fine pollen grains in water, he gave the first proof of the existence of atoms, the smallest chunks of matter. The reality of atoms was still disputed by many physicists of the time as they are too small to see, even under a microscope. Einstein also accounted for the photoelectric effect by assuming that light itself came in quantum chunks. The quantum of light was only generally accepted and named the photon in the 1920s. When quantum mechanics was formulated in 1925-27, it became clear that all nature's interactions are digital, with the unit chunk being Planck's constant of "action". Heisenberg's uncertainty principle limits the accuracy of simultaneous measurement of conjugate pairs to the Planck action unit. Nature is strictly digital.

Shannon's Sampling Theorem of 1940 was the great enabler of conversion from analogue media to digital media. It says that, by sampling at twice the maximum frequency of a signal, you can extract *all* the information in the signal; no quality is lost. Measure the wave height once for each of the fastest wiggles up or down of the wave, and you know everything there is to know about the wave. This astonished many people at the time and still seems highly counterintuitive — how can you just measure certain intermittent points in a signal, ignoring all the rest of it, and still have *all* the information that the signal contains? Well, it seems that information is naturally digital and quantised into bits, that is just how it is.

For high quality digital sound recording, samples of sound intensity are measured by a microphone 44,100 times per second (Hertz). Humans can only hear frequencies up to about 16,000 Hertz, and Shannon's sampling theorem assures us that sampling at twice the maximum frequency extracts all the information, so recording 44,100 samples per second provides perfect quality to human ears.

Strangely, although our world seems to have become more and more digital rather than analogue, mathematics still continues to assume the reality of the continuum — the idea that you can carry on dividing something as long as you like, ad infinitum. In computers, clever programming allows you to use 'real' numbers with as many decimal places as you need, but the underlying code is actually breaking everything down into individual bits. The real world is digital, not continuous.

> Arbitrary precision requires an unlimited memory, and this is unlikely to exist in a finite universe. Even if the universe is unlimited, it seems unlikely that

> we can collect arbitrarily large parts of it into an organized memory structure, and even if we grant the availability of unlimited memory, it still seems unlikely that we can have an unlimited sequence of operations, each guaranteed to be totally free of error. Thus we are questioning that the mathematicians' continuum and real number system reflect executable operations.
> Rolf Landauer, *Information is Inevitably Physical*, in *Feynman and Computation*

Gabor's Theory of Communication

Dennis Gabor was the first to apply the mathematical machinery that had been developed for quantum mechanics to the analysis of signals, particularly sound. In his 1944 paper *The Theory of Communication*, he showed that not only are signals inherently digital, made up of elementary units that he proposed to call *logons*, but also that Heisenberg uncertainty is far more commonplace than just being a feature of the quantum world.

> Hitherto communication theory was based on two alternative methods of signal analysis. One is the description of the signal as a function of time; the other is Fourier analysis. Both are idealizations, as the first method operates with sharply defined instants of time, the second with infinite wave trains of rigorously defined frequencies. But our everyday experiences – especially our auditory sensations – insist on a description in terms of both time and frequency. In the present paper this point of view is developed in quantitative language. Signals are represented in two dimensions, with time and frequency as co-ordinates. Such two dimensional representations can be called "information diagrams," as areas in them are proportional to the number of independent data which they can convey. This is a consequence of the fact that the frequency of a signal which is not of infinite duration can be defined only with a certain inaccuracy, which is inversely proportional to the duration, and vice versa. This "uncertainty relation" suggests a new method of description, intermediate between the two extremes of time analysis and spectral analysis. There are certain "elementary signals" which occupy the smallest possible area in the information diagram. They are harmonic oscillations modulated by a "probability pulse." Any signal can be expanded in terms of these by a process which includes time analysis and Fourier analysis as extreme cases.
> Dennis Gabor, *The Theory of Communication*, 1944

The example of a sound signal is easiest to understand. Hearing a sound relies on detecting the up and down reversals of the sound wave. Time is obviously required for this, but so is frequency — if the wave doesn't cycle up and down there is nothing to hear, and if you don't have sufficient time, you can't measure a frequency. A single variable is really doing double duty. You count time units to measure time as duration. You count how many changes occur *within* a time unit to measure frequency.

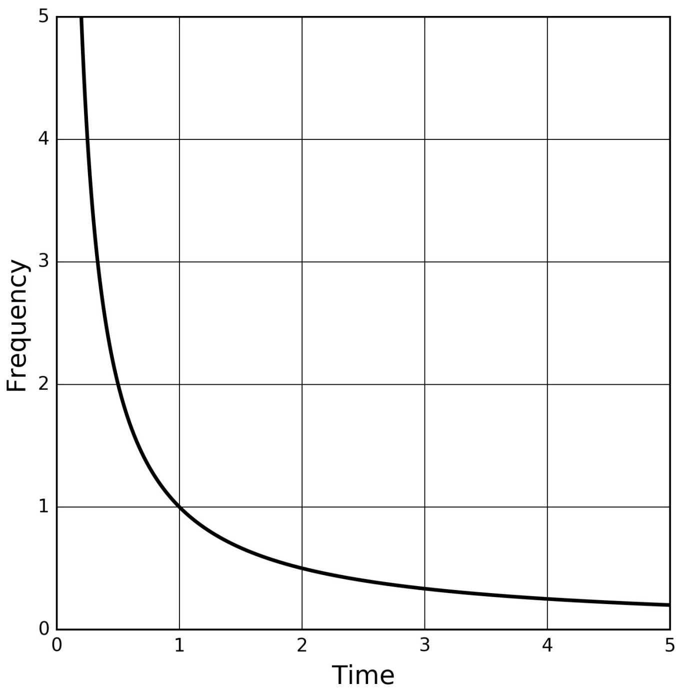


Fig. 10.1 Time=1/frequency. Frequency=1/time.

Time and frequency are reciprocals of each other. If we plot this relationship as a graph we get the hyperbola of Fig. 10.1. The Fourier method allows the description of a signal in either the time, or frequency picture, but when we hear a sound, we hear a frequency which also has a duration in time. How can this be? Gabor discusses this apparent contradiction:

> Though mathematically this [Fourier] theorem is beyond reproach, even experts could not at times conceal an uneasy feeling when it came to the physical interpretation of results obtained by the Fourier method. After having for the first time obtained the spectrum of a frequency-modulated sine wave, Carson wrote: "The foregoing solutions, though unquestionably mathematically

> correct, are somewhat difficult to reconcile with our physical intuitions, and our physical concepts of such 'variable-frequency' mechanisms as, for example, the siren."
>
> The reason is that the Fourier-integral method considers phenomena in an infinite interval, sub specie aeternitatis, and this is very far from our everyday point of view. Fourier's theorem makes of description in time and description by the spectrum (frequency), two mutually exclusive methods. If the term "frequency" is used in the strict mathematical sense which applies only to infinite wave-trains, a "changing frequency" becomes a contradiction in terms, as it is a statement involving both time and frequency."
>
> . . . speech and music have for us a definite "time pattern," as well as a frequency pattern. It is possible to leave the time pattern unchanged, and double what we generally call the "frequencies" by playing a musical piece on the piano an octave higher, or conversely it can be played in the same key, but in a different time. Evidently, both views have their limitations, and they are complementary rather than mutually exclusive.
> Dennis Gabor, *The Theory of Communication*, 1944

The Time-Frequency Diagram

Recognising the necessity of the time and frequency pictures, Gabor introduces the time-frequency diagram, which he called an information diagram, plotting time and frequency as orthogonal coordinates. He goes on to show that there is a smallest measurable quantity, corresponding to the smallest possible rectangle in the time/frequency diagram, and proposed that it be named a ‘logon’. This fundamental unit of information has also been called a ‘Shannon’, but is now known as a ‘bit’, a shortened form of ‘binary digit’.

> . . . for every type of resonator a characteristic rectangle of about unit area can be defined in the time/frequency diagram, which corresponds to one "practically" independent reading of the instrument. In order to obtain their number, we must divide up the (time * frequency) area into such rectangles.
>
> . . . any signal can be expanded into elementary signals in such a way that their representative rectangles cover the whole time-frequency area, . . . Each of these areas, with its associated datum, represents, as it were, one elementary quantum of information, and it is proposed to call it a logon.
> Dennis Gabor, *The Theory of Communication*, 1944

The time and frequency domain representations are two unobtainable extremes, any real signal is a compromise between the two, a sound has frequency *and* extends in time. The product of the uncertainties in measuring the time and the frequency are of the order of unity, given by the formula

$$\Delta t \times \Delta f \approx 1$$

where Δt and Δf are the uncertainties in the definitions of the epoch t and the frequency f of an oscillation. The uncertainty defines the minimum area of the "Heisenberg box" representing the smallest measurement of 1 unit. The Heisenberg box can be short and fat, or long and thin, but it must always be of unit area.

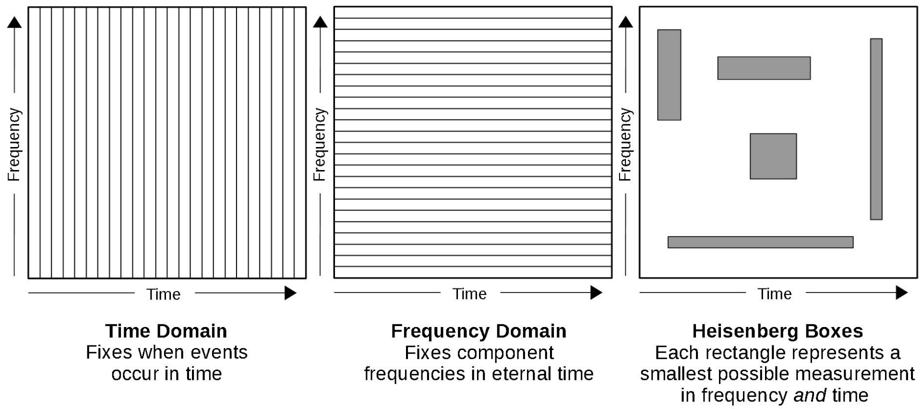


Fig. 10.2

1) Time domain data, 2) Frequency domain data, 3) Data in both domains

Fig. 10.2 shows from left to right: 1) how the time domain distinguishes when events occur in time, 2) how the frequency domain identifies the values of component frequencies in eternal time, and 3) various ways that a smallest measurement, can be made in frequency *and* time.

> . . . it is now evident that physical instruments analyse the time-frequency diagram into rectangles which have shapes dependent on the nature of the instrument and areas of the order unity, but not less than one-half. The number of these rectangles in any region is the number of independent data which the instrument can obtain from the signal, i.e. proportional to the amount of information.
> . . . We may now ask what it is that prevents any instrument from analysing the information area with an accuracy of less than a half unit. The ultimate

reason for this is evident. We have made of a function of two variables, – time *or* frequency – a function of two variables – time *and* frequency. This might be considered an artificial process, but it must be remembered that it corresponds very closely to our subjective interpretation of aural sensations.
. . . as a result of this doubling of variables we have the strange feature that, although we can carry out the analysis with any degree of accuracy in the time direction or the frequency direction, we cannot carry it out simultaneously in both beyond a certain limit.
Dennis Gabor, *The Theory of Communication*, 1944

Musical Uncertainty

If you ever use a guitar tuner, you will notice that the device takes some time to settle onto a frequency value after you have plucked a string. Time is needed to count the number of cycles of the sound wave in order to give a frequency figure, and the more time that is allowed for the measurement, the more accurate it will be. You reduce the uncertainty in the frequency by allowing more time to measure it: **frequency measurement requires time**. Conversely, any measurement of time relies on counting some frequency, such as the swings of a pendulum, or the oscillations of atoms: **time measurement requires frequency**.

Time-Frequency Representations

A music file stored on a computer is just a long list of numbers describing air pressure measurements, with 44,100 samples for every second of the recording. You can store exactly the same information in Fourier dual form. Instead of having a list of numbers strung out in regular time steps, you can transform the data from the time domain to the frequency domain, creating a list of numbers that are laid out in regular frequency steps. Sound processing software often includes a function called *spectral analysis*, that does just this. In the dual, frequency domain, or spectral view, the frequencies of musical notes are immediately obvious, but time information about when the notes were played is gone. To find out *when* a note is played, you would have to reverse-transform the data back to the time domain.

Fig. 10.3 shows how each of 16 separate reference frequencies behaves when allowed to wiggle for the same duration of 16 time steps. We use just 16 frequencies and 16 time steps for simplicity. Do not confuse the frequencies, or pitches, of notes you can hear with these unchanging reference frequencies that mark each step on the vertical frequency axis of Fig. 10.3. The grey lines show how the sine value of each frequency varies at different times across the page, and the black lines represent the cosine values of each frequency. Sines and cosines wiggle up and down between +1 and -1. When either the sine or the cosine is crossing the 0 point in the middle, the other one is at a maximum or minimum, +1 or -1. The sine and cosine are 1/4 of a cycle out of phase with each other, and one is the rate of change of the other. They provide two separate channels for information. These are basic properties of sines and cosines.

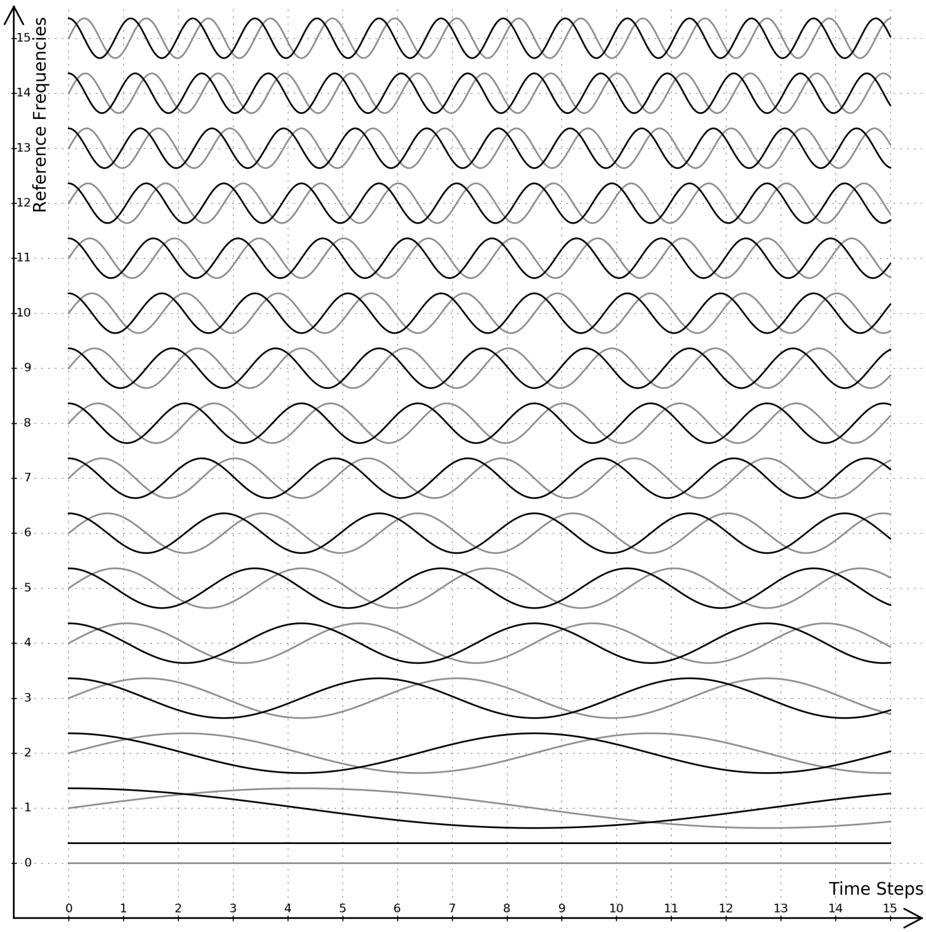


Fig. 10.3 16 reference frequencies oscillate during 16 time steps.

The reference frequencies displayed in Fig. 10.3 have fixed amplitudes, and carry no information in themselves, they simply keep wiggling for ever in the same fashion. But by setting different amplitudes for the sine and cosine of each reference frequency, the frequency domain as a whole can represent all the information in the time domain. A Fourier transform of the time domain data calculates the required frequency domain amplitudes. Each time step only needs one piece of information, but each frequency step requires two — a real cosine value and an imaginary sine value. Reference frequencies are used to probe the sound recording, detecting how much of each frequency is present, and at what phase. The phase of a frequency can be found from the relative proportions of its sine and cosine components.

Rather magically, an entire sound recording in the form of its time domain sample values, can be recreated, or 'synthesised' from the frequency domain picture, by just giving the calculated sine and cosine frequency amplitudes time to wiggle. As time unfolds, the output amplitude at each time step is generated by adding together the sum of *all* the reference frequency amplitudes at that instant. Some of the wiggling waves will be above their average and others below. It is the cooperation of all of them in 'superposition' that performs the magic. Just as you need *all* the frequencies to give the value at one time step, you need the information from *all* the time steps to go the other way and calculate the pair of amplitudes for each frequency step. Information that is *localised* in one domain is *globalised* in the other.

The time domain picture and frequency domain picture are dual versions of the same information, and the mathematical processes of converting one to the other, Fourier transform and reverse Fourier transform, are almost identical. Because what is local in one is global in the other, if you change a single frequency's magnitude or phase in the frequency view, you will affect, and may even ruin, the whole sound playback through all of time. Conversely, if you change a single value in the time view, all the frequency values will be affected. This property of information being representable in a local or global form is also characteristic of the hologram, in which a single pixel contains information about the whole picture, the mathematics of the hologram is very similar to the Fourier transform. It was Dennis Gabor who invented holography, and it was for this that he won the 1971 Nobel prize for physics. The local↔global property of information also arises in physics because of the fundamental importance of Fourier dualities in quantum mechanics. We look at this in more detail in Chapter 11.

A Simple Example of Fourier Analysis

For this highly simplified example we will use a waveform that might be from a sound recording, but could be any other time varying data. We will sample the waveform at 16 steps in the time domain, and Forward Fourier transform this data into 16 values in the frequency domain. We will then Inverse Fourier transform the frequency domain data back into the time domain. If Fourier was correct, we should get back to the same sampled data values that we started with.

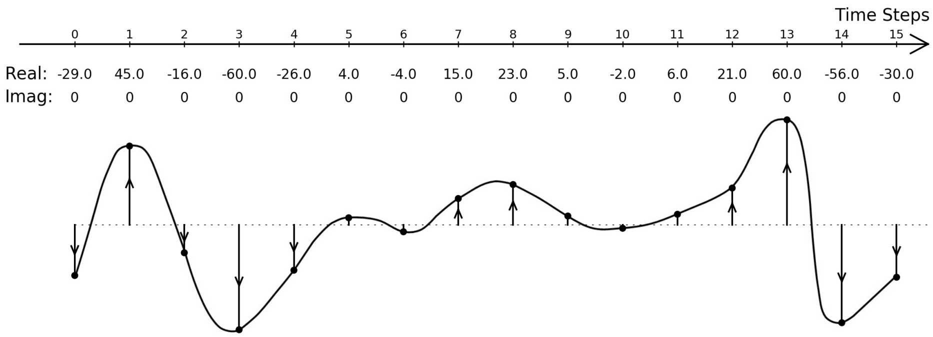


Fig. 10.4 The height of a sound wave is measured (sampled) at 16 time steps. These real values are recorded above, the imaginary values are all zero.

A fragment of the recorded sound signal is displayed as the curve in Fig. 10.4. The height of the wave is sampled at 16 steps in time, arrows pointing up giving a positive value, while downward arrows give a negative value, and these values are recorded as real values at the top of Fig. 10.4. A Fourier transform requires two channels of information, real and imaginary, in each domain. In the time domain, we only need the real values, the imaginary values all remaining zero. In the frequency domain, real values are cosines, and imaginary values are sines.

We will also use 16 frequency steps in the frequency domain, and must calculate 16 x 16 = 256 combinations. You can look at this time↔frequency diagram as a 16 x 16 matrix of numbers, with each cell of the matrix corresponding to a particular combination of a time step and reference frequency. Each of the 16 reference frequencies is used to probe the time domain data, finding out how well it correlates with the signal waveform over the whole duration of 16 time steps, and thus how much amplitude that frequency should have in the frequency domain representation of the data.

We use these formulae:

Forward Transform: $$F[f]=\frac{1}{16}\sum T[t]\mathrm{e}^{-i2\pi ft/16}$$

Reverse Transform: $$T[t]=\sum F[f]\mathrm{e}^{i2\pi ft/16}$$

The 16 time domain sample values are stored in $T[t]$ where t is the sample number. Similarly, the frequency domain values are stored in $F[f]$ where f is the frequency number. The summation sign Σ means add up what is to the right of it for all 16 time steps t, or frequency steps f. At the right of each formula is a complex exponential, and as we saw in Chapter 4, this is another way of representing a real cosine value plus an imaginary sine value. The product of frequency and time, ft, in each exponent, keeps track of the phase angle of the combined real and imaginary parts for each of the 256 cells in the transformation matrix. The product of the value at a step, with the value of the complex exponential at that step, measures their correlation, and gives the weighting for that cell in the matrix.

Forward Fourier Transform

To calculate the value for each cell in the matrix we multiply two values together: the sample value we have just found at that time step, and the instantaneous amplitude of the cosine of the reference frequency at that time step. The reference frequency amplitude will be positive or negative, depending on whether its wave is above or below its average height at that instant in time. We add up all the amplitudes we get for each of the 16 time steps across the matrix for a particular reference frequency, divide by 16 to get the average value, and put the result in the Cos column for that frequency on the left of Fig. 10.5. Staying at the same frequency, we repeat this procedure, but use instantaneous sine values of the reference frequency at each time step, and put the results in the Sin column.

We can see the results of all this calculation on the left of Fig. 10.5, the 16 values of our data have been transformed into 16 cosine and sine pairs in the frequency domain. The curve that is displayed for each frequency in Fig. 10.5 is now the sum of the cosine and sine curves, it is a new sinusoidal wave of the same reference frequency, but with its amplitude adjusted, and its phase shifted. The cosine and sine amplitudes fix the amplitude and the phase of the combined waveform for that frequency. You can see that some frequencies now make bigger contributions than others. In this example frequencies 3 and 4 have large amplitudes, but frequencies 2 and 14 are almost inactive, they carry little information.

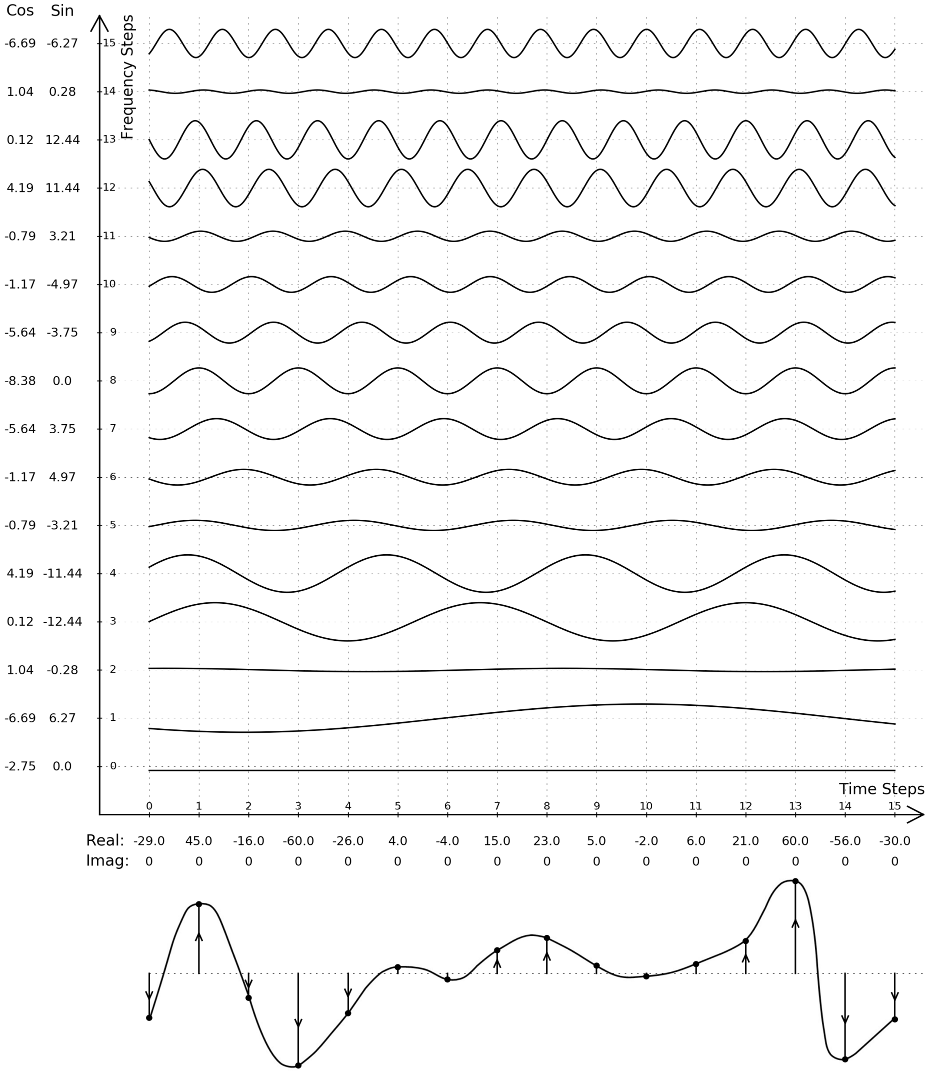


Fig. 10.5 Forward Fourier Transform.
Time domain sample data (bottom) is converted into frequency domain data as amounts of the Sine and Cosine phases (left).

Notice that the cosine values for frequencies 1 and 15 are the same, -6.69, and so are the values for 2 and 14, 3 and 13, etc. The same is true for the sine values, except that each pair of values are opposite in sign, so that for example, frequency 11 is 3.21, but frequency 5 is -3.21.The frequency domain folds the information into two parts, one having positive frequencies and the other having negative frequencies. In our example, the middle frequency, 8, marks the change between positive and negative frequencies. A negative frequency seems to make no sense, as it requires time to run backwards, and this is physically impossible. But when time is measured by rotation, it can be reversed simply by reversing the direction of rotation. Negative frequencies are an essential component of Fourier analysis, and we will see their importance in physics later.

Reverse Fourier Transform

Going in the direction of a reverse Fourier transform, all the frequencies have a say in what the sample value should be at each time step. To make a reverse, or synthesis Fourier transform, we work horizontally through the time domain, step by step. We add up all the positive and negative contributions from the modified frequency curves vertically above each time step in Fig. 10.5, and this overall sum gives the real value for that time step. As shown in Fig. 10.6, we end up back with our original sample values — Fourier was right!

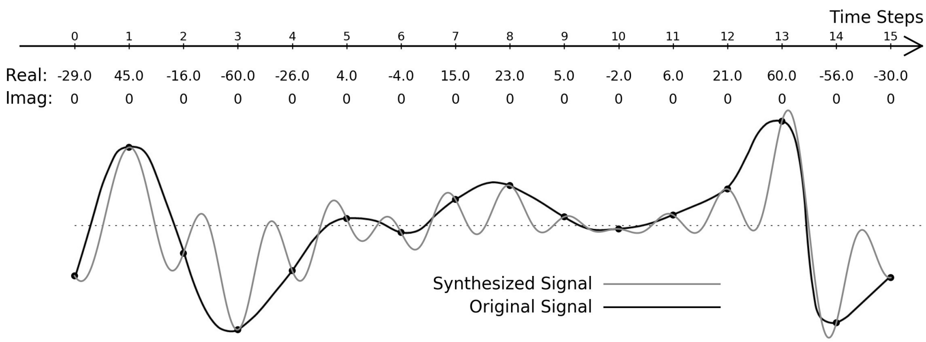


Fig. 10.6 Reverse Fourier Transform.
Sample point values are reconstructed from the frequency domain data.

The grey curve of Fig. 10.6 is synthesized from the frequency domain data by adding together all the frequency contributions continuously in time, rather than only at the time sample steps. You can see that it passes correctly through the sampled points on the original curve, but varies quite wildly in between them. Using more samples would improve the fit, and Shannon's sampling theorem tells us that we must sample at twice the maximum frequency contained in a signal to extract all the information it carries.

We have used only real sample values in this example, but a more complete version of the process above is called a Complex Discrete Fourier Transform and uses a pair of values for both the frequency domain data and the time domain sample data. Professionals in the field of digital signal processing usually work with complex numbers as the two compartments of a complex number naturally hold a cosine and sine pair, or, if you prefer it, an amplitude and phase.

> In spite of its abstract nature, the complex Fourier transform properly describes how physical systems behave. When we restrict the mathematics to be real numbers, problems arise... In the world of mathematics, the complex Fourier transform is a greater truth than the real Fourier transform. This holds great appeal to mathematicians and academicians, a group that strives to expand human knowledge, rather than simply solving a particular problem at hand.
> Steven Smith, *The Scientist and Engineer's Guide to Digital Signal Processing*

Perfect Wave Packets

A wave packet is a collection of waves of slightly different frequencies whose mutual interference conspires to give the appearance of a solid wave. In quantum mechanics, particles are modelled as wave packets of the same Gaussian shape that Gabor showed to be optimum for packing the most information into a signal. Fig. 10.7 illustrates how the trade off between time and frequency works.

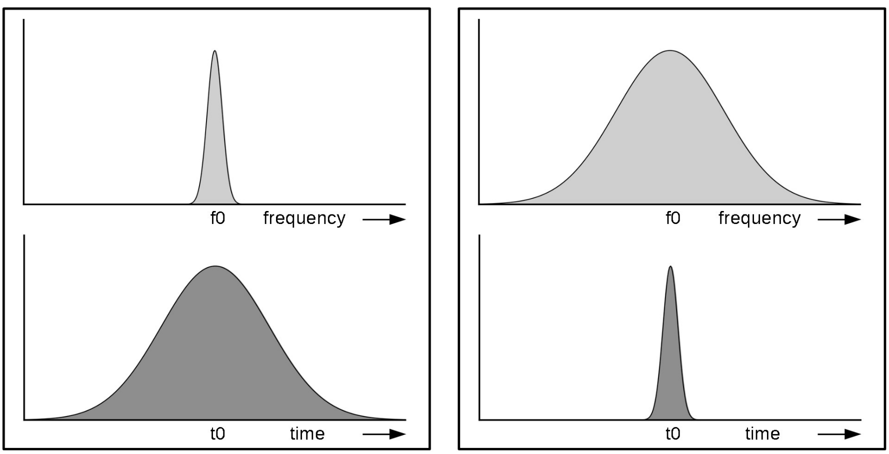


Fig. 10.7 Left: sharp frequency wave packet. Right: sharp time wave packet.

To get a sharp measure in the frequency domain for a wave packet, you need a wide span of time. To get a sharply defined time you need a wide bandwidth of frequencies. The time/frequency plane is used most efficiently when the Heisenberg boxes are square, making equal use of time and frequency, and wave packets take the form of bell shaped (Gaussian) probability curves of equal sharpness in time and frequency, as shown in Fig. 10.8. All the information in an ideal signal is concentrated within the time and frequency range of the wave packet, centred around t0 and f0. Within this range all the component waves interfere constructively to create the packet, whereas outside this range, the component waves interfere destructively, and the result is nothingness. The Gaussian wave packet that is used in quantum mechanics to describe a particle, is not one wave, but a packet of many waves, superposed on each other to give the appearance of a particle. We will look at elementary particles and their quantum nature in Chapters 11 and 13.

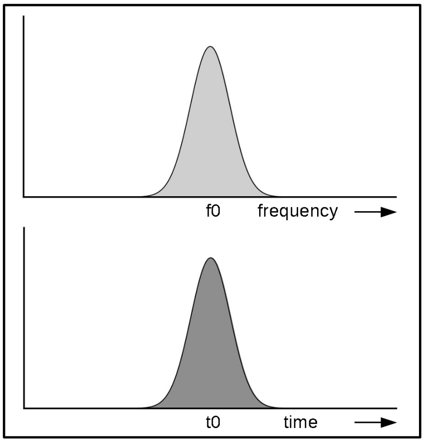


Fig. 10.8 A Gaussian wave packet has equal spread in time and frequency.

Causality

If you managed to follow the Fourier transform example above, you may have noticed that we had to take all of 'eternity' in the time view into account in order to calculate a single value in the frequency view.

> ... the principle of causality requires that any quantity at an epoch t can depend only on data belonging to epochs earlier than t. But we have seen that we could not carry out the expansion into elementary signals exactly without taking into consideration also the "overlap of the future." In fact, strict causality exists only in the "time language"; as soon as we use frequency as an additional reference the sort of uncertainty occurs which in modern physics has often been called the "breakdown of causality."
> Dennis Gabor, *The Theory of Communication*, 1944

Quantum mechanics tells us that many of the important quantities that are measured in physics are on one 'side' of a Fourier duality. Length is Fourier dual to momentum, energy is Fourier dual to time, and angular momentum is Fourier dual to phase. Somehow, Nature keeps the information within each of these dualities synchronised across the dual domains, and this must require a lot of processing — this may account for some of the extreme processing power in Nature that the Margolus-Levitin theorem reveals. The breakdown of causality that Gabor described, is also telling us that there is much we fail to understand about how causality really works — the future must play a role, as well as the past. As we have seen, Fourier transforms require time reversal in the form of negative frequencies, and we will see in Chapter 13 that time reversal is an inherent part of the Dirac equation that describes the internal structure of an electron.

The Laplace Transform

Besides the Fourier transform, there are many other mathematical transforms that are useful for various purposes. A more powerful extension of the Fourier transform is the Laplace transform, and it is a favourite tool for engineers in Digital Signal Processing, DSP, applications. In the Fourier transforms between time and frequency domains shown above, the exponents in the complex exponentials e^{ift} and e^{-ift} are purely imaginary, because f and t are just real numbers, so the exponents ift and $-ift$ are both purely imaginary. As we have seen, these can only produce sinusoids and rotations around the unit circle, they cannot scale things up or down in the way you might expect from 'exponential' behaviour. A Laplace transform is similar to a Fourier transform, but uses exponents that are complex numbers which contain real as well as imaginary components, and it is the real parts that allow the exponentials to scale things up or down in the familiar manner of exponential increase or decrease. An example where this is useful to engineers, is in predicting unstable feedback in an amplifier, like the unpleasant, exponentially increasing howlround that can occur when a microphone is put too close to a loudspeaker.

The Laplace transform is also important in quantum mechanics, and was much used by Schrödinger. (Because of the rather subtle difference between the Fourier and Laplace transforms, people often call them both Fourier transforms.) The Laplace transform is described in Chapter 32 of Steven Smith's book and online guide, *The Scientist and Engineer's Guide to Digital Signal Processing*. Smith shows how signals can be described on a complex plane called the *s-plane*. Real exponentials are represented horizontally, increasing to the left, and decreasing to the right, while imaginary exponentials are represented vertically, with positive frequencies (forward time) going up, and negative frequencies (reverse time) going down. The zero origin at the centre of the complex plane represents where time is stopped, with no tendency of the signal to exponentially increase or decrease, and any other location on the s-plane can identify for DSP engineers where a phenomenon like howlround may occur.

For anyone who, like me, has more of a software engineer's mindset than a mathematician's, a few lines of code is much easier to understand than an equation — you can step through the code in your mind and get an idea of what is going on. Remember also that standard mathematics is wedded to the idea that continuous quantities are possible, even though we know that Nature is strictly digital, so a coder's view has more validity than a mathematician's.

Steven Smith's book and online guide *The Scientist and Engineer's Guide to Digital Signal Processing* is a wonderful resource. It is very clearly written, and procedures like the various types of Fourier transform are not only given as equations, but also laid out as BASIC code that is easy to follow, and explained with the help of excellent diagrams. I recommend it very highly for anyone seeking to understand the Cosmic Computer's operation. After all, we should acknowledge that Nature has known and exploited our clever DSP techniques for information processing all along…

11
Quantum Mechanics

Planck

In 1900, Max Planck, in what he later called ‘an act of desperation’, introduced the quantum into physics. He did not even believe in the reality of atoms as discrete units of matter, but the only way he could get his formula for the radiation given off by a hot body to agree with the experimental data was to assume that energy was emitted in fixed sized units. He found that the amount of energy in each unit must depend on the frequency of the radiation according to the now famous law

$$E = hf$$

E is the energy, h is Planck's constant and f is the frequency

Planck’s constant is far too small to be noticeable in the everyday world. It has the value of 6.63×10^{-34} JouleSecond, and the dimensions of action, ML^2/T, as we saw in Chapter 5. It is the fundamental currency of the Universe, but Planck did not pursue the implications of his quantum unit any further, he only sought some way of getting rid of it, but failed.

For many years, Einstein was almost alone in taking the quantum idea seriously, and by following its logical implications, he produced some of his greatest work. He championed the idea that if energy was emitted in units it must also travel as units (which were later called photons), as well as being a wave. Although light was known to travel as electro-magnetic waves, Einstein, unlike Planck, was able to embrace the apparent contradiction of light having a dual wave↔particle nature. He explained the photoelectric effect. He accounted for Brownian motion, and he proved the existence of atoms. Nevertheless, despite Einstein’s efforts, the existence of photons was denied by most physicists until the 1920s.

Ernest Rutherford’s experiments of 1911 showed that the atom has a tiny positively charged, and heavy nucleus, surrounded by a cloud of light, negatively charged electrons. According to theory this arrangement should have been completely unstable, the electrons being pulled into the nucleus by the electrical attraction between opposite charges, or if they were in orbital motion, the acceleration they experienced should make them emit light, and lose energy. In 1912, Niels Bohr came up with a model of the atom as a nucleus with electrons in orbit around it, with some new, unknown physics to keep the orbits stable and prevent light emission. Bohr's orbits had to be at specific levels set by multiples of Planck's quantised action unit, fixed steps of angular momentum, with an absolute minimum level that ensured the atom's stability. An electron that changes levels emits or absorbs light with a frequency proportional to the energy jump between the levels. The frequency of visible light is also its colour, so Bohr's model could explain the existence of the mysterious coloured lines that are observed in the spectrum of light radiated by hot atoms, each element in the periodic table having its own signature pattern of spectral lines.

Matrix Mechanics

Just as it was experimental data on the spectrum of radiation given off by a hot black body that had baffled Planck and forced him to go digital, it was again the rules governing the behaviour of the spectral lines of atoms that baffled Werner Heisenberg in 1925, and prompted him to start multiplying arrays of numbers, in what seemed to him like a mathematically dubious procedure, that nevertheless seemed to give the right answers. Heisenberg showed his new method to Max Born, who became intrigued, and spent several days struggling to understand Heisenberg's method before suddenly realizing that Heisenberg's arrays of numbers were matrices, and that Heisenberg's dubious procedure was just the non-commutative multiplication of matrices that we saw in Chapter 4. Heisenberg's method was the first version of quantum mechanics, and it is now called matrix mechanics.

A distinctive feature of quantum mechanics is that some of its variables appear in conjugate pairs, like length↔momentum, and energy↔time. (Note that the word conjugate here has a different meaning than it has when used to describe the complex conjugate of a complex number). The conjugate variables of quantum mechanics are entwined in a Fourier duality relationship, similar to what we saw in the context of sound in Chapter 10. But whereas the time↔frequency duality of sound is a simple reciprocal relationship of time↔1/time, in quantum mechanics the situation is more complicated, as the conjugate pairs are not simple reciprocals of each other, and the quantities are represented by complex numbers in matrices. Instead of just the bare number 1 as the minimum product of a conjugate pair, you get 1 Planck unit of action, which has dimensions of Mass x Length^2 x Time^{-1}. Here again are the dimensional relationships of these conjugate quantities to Planck's constant unit of action.

$$\frac{ML^2}{T^2} \times T \quad = \quad \frac{ML^2}{T} \quad = \quad \frac{ML}{T} \times L$$

Energy x Time = ACTION = Momentum x Length

Born found the matrix formula that connects a particle's position q and momentum p to be $pq - qp = (ih/2\pi)I$, where I is a unit matrix. If you have a system of many particles, you can multiply the position q of one particle and the momentum p of another in any order you like, it makes no difference; $pq = qp$, so $pq - qp = 0$. It is only when the p and q refer to the same particle that non-commutativity applies and Planck's unit of action h pops up in the 'imaginary' form ih. The matrix formalism automatically takes care of this. It seems that switching from the position-first view of a particle to the momentum-first view involves a displacement in some orthogonal dimension — the position information and the momentum information exist in separate realms, and there is a sort of Planck-sized magic roundabout that must be negotiated to go from one to the other.

Wave Mechanics

While matrix mechanics gave accurate results for some otherwise intractable problems, it was totally abstract, and ignored the wave nature of particles that Louis de Broglie had introduced in 1923. De Broglie had postulated that if a light quantum could be either a wave or a particle, the same should apply generally, so that an electron, for example, should behave like a wave as well as a particle. He reasoned that only electrons of certain fixed wavelengths would fit around an atomic orbit, and that Bohr's stable electron orbits are those that allow standing electron waves. The wavelength de Broglie calculated for a particle is given by the equation:

$$\lambda = h/p$$

λ is the wavelength, h is Planck's constant and p is the momentum

In 1925, two different experiments showed that beams of electrons could be diffracted by crystals and thin gold foils, and the results agreed with de Broglie's wavelength equation. G. P. Thomson was one of the experimenters who won a Nobel prize for this proof of the wave nature of the electron, his father J. J. Thomson having earlier won a Nobel for discovering the existence of the electron as a particle.

The behaviour of waves is described mathematically by a wave equation, and towards the end of 1925, Erwin Schrödinger, disgusted by the abstraction of Heisenberg’s matrix mechanics, started searching for a wave equation. In 1926, he introduced a new version of quantum mechanics, *wave mechanics*, which he later showed to be equivalent to matrix mechanics, but framed in different mathematics. Schrödinger’s wave equation describes how a wave function varies in time and space, and because this mathematics was more familiar and easier to use than Heisenberg's matrices, wave mechanics quickly became the favourite tool of the new quantum mechanics. Max Born found that squaring the wave function gave the probability of finding a particle at a particular location in space, handily getting rid of the imaginary parts of the complex wave function. We saw the trick of how squaring a complex number makes it real in Chapter 4.

Schrödinger had hoped to find a form of quantum mechanics that was more visualisable than matrix mechanics, but it quickly became apparent that there were problems of interpretation in his wave picture. Schrödinger's wave function could take on complex values, unlike the purely real values of normal wave equations, and if more than one particle was involved, the waves travelled in a multi-dimensional configuration space rather than normal 3D space. Schrödinger found that he could model a particle as a wave packet, a superposition of waves of different frequencies that interfere with each other constructively at the particle location and destructively elsewhere, but he could find no way of preventing the wave packets from spreading out with time. De Broglie's original concept of a particle riding like a surfer on matter waves seemed to be ruled out.

Schrödinger’s wave equation models the evolution of a quantum state vector, or wave function, in multi-dimensional, complex, *Hilbert space*. Schrödinger evolution in Hilbert space is unitary, the mathematics is confined within the scale of a unit, like traffic confined to a fixed size of roundabout — the size of a quantum state vector doesn’t matter, all that counts is the direction in which it points in Hilbert space. A roundabout can have many roads leading in and out of it in different directions, and these may be thought of as separate dimensions. The *unitary* internal workings of quantum mechanics seem to be concerned with the choice of route among possible direction dimensions.

Operators

Quantum mechanics asserts that *all* the information about a physical entity is contained in its wave function, represented as a state vector in Hilbert space. Only the 'direction' of a vector in Hilbert space has physical significance, its magnitude is irrelevant. Real information can be extracted from a product of vectors because Hilbert space is formally defined as having a positive-definite inner product which gets rid of imaginary quantities. To make predictions about something physical, you have to apply an *operator* to the wave function, and the result is a fixed real expectation value of the operator. The simplest operator is the unit operator 1_{op} which produces no change. The operator representing position x is simply the position vector x, but some, like the momentum operator $-i\hbar d/dx$, are differential operators which represent the rate of change of one quantity with another. (d/dx describes the rate of change of something when x changes). Notice that the imaginary unit i appears as soon as changes in time are considered. The fact that there is no operator for time is another mystery of quantum mechanics — time just seems to be special, it has privileged access to wave functions everywhere, despite the non-existence of a universal time according to standard physics. When differential and ordinary operators are applied in succession, the order matters, they do not commute, and this is another manifestation of the non-commutativity that we have already seen for position↔momentum and time↔energy.

If we let $A_{op} = d/dx$ and $B_{op} = x$ we find that

$$A_{op}B_{op} - B_{op}A_{op} = 1_{op}$$

whereas the right hand side should be zero if A and B were ordinary numbers. This strange behaviour of differential operators is a consequence of Leibniz's theorem which gives the rule for differentiating a product of two quantities. We can simply multiply a product fg by an ordinary number C to get Cfg, but using D to represent the differential operator d/dx, Leibniz's theorem tells us that

$$D(fg) = fDg + gDf$$

Fig. 11.1 shows why the differential, D, must be made infinitesimally small for Leibniz's theorem to hold true. When you apply the differential operator D to find the infinitesimal increase of the grey area measured by the product fg, Leibniz's formula adds on only the top area, gDf, and the right side area, fDg. The small black area DgDf at the top right is left out, and this is the 'error' in Leibniz's theorem that only disappears when the change produced by D is infinitesimally small, and the strips added to the top and right hand side become infinitely thin — the black area DgDf, a double differential, effectively disappears. This is the essential trick of differential calculus — sweeping very small things under the carpet.

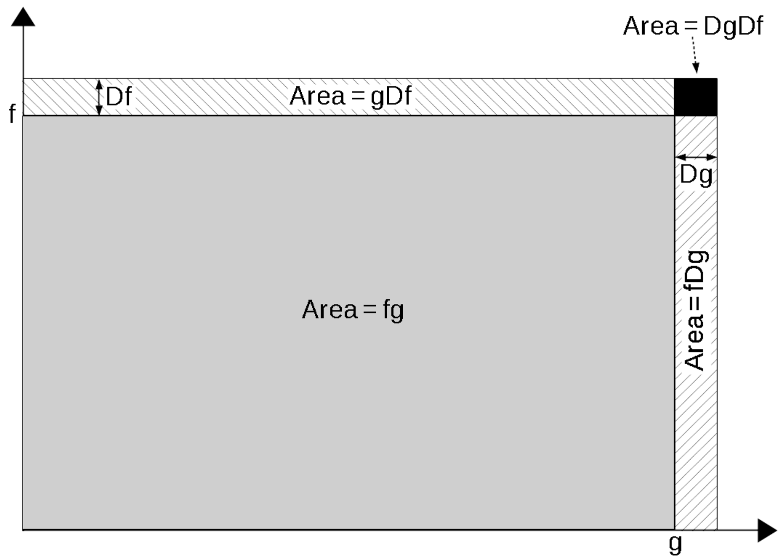


Fig. 11.1

The product of two differentials can be made to disappear.

Leibniz's theorem only works because differential calculus makes use of infinitesimal quantities. Calculus relies on using infinitesimal quantities to perform the trick of being small enough to be negligible, whilst at the same time having a real effect. They work 'in the limit', where a quantity almost disappears, but just enough still remains when it is needed to do something useful. An infinitesimal squared, something tiny times something tiny, is so small as to conveniently disappear. Many mathematical tricks rely on this idea of the very small being allowed to break the normal rules. Another example is Riemannian geometry, which is used to describe the gravitationally induced curvature of space-time, and relies on the idea that, at a small enough scale, below the 'limit', curvature disappears and space-time is flat. One might think that there is a built-in contradiction here — how can mathematics describe curved space-time with a procedure which assumes space-time has zero curvature at small enough scales?

Dual Pictures

The wave function can have more than one representation. So, for example, you can represent the wave function in normal coordinate space as $\Psi(x)$, where x is a length, or you can use its Fourier transform $\Phi(p)$ in momentum space, where p is a value of momentum. Because $\Psi(x)$ and $\Phi(p)$ are bound together as exact Fourier duals of each other, either function gives a complete description of the quantum state of the system, and any equation involving one of the functions can be written in terms of the other. Physicists routinely switch between the coordinate space and momentum space pictures, using whichever is most convenient for the task in hand, just as engineers, and your smartphone, flip information back and forth between the Fourier dual time and frequency domains when processing signals. And, just as more precision in time means less precision in frequency, a particle with an exactly defined position in coordinate space will be infinitely spread out in momentum space, and a particle with a precisely defined momentum could be anywhere in normal space. This means that if you know that a particle is absolutely still, with zero momentum, it could be anywhere in the Universe!

As we noted above, Schrödinger's wave mechanics and Heisenberg's matrix mechanics are equivalent to each other, only using different mathematics. They also allow physicists to switch between the Schrödinger and Heisenberg 'pictures' of how a quantum state (the state vector or wave function) evolves with time. In the Schrödinger picture the quantum state evolves but the operators never change. In the Heisenberg picture, it is the other way around, a quantum state does not change with time, it is only the operators that are required to reveal its contents that vary with time. There is also the Dirac picture in which both states and operators vary with time — this is also called the interaction picture and is the basis of quantum field theories.

Feynman's Least Action Quantum Mechanics

We looked at the least action principle in Chapter 6, and saw how Nature seeks perfection in all its workings, finding the optimum path for reality among all the possible paths. This is done mathematically by continually minimising the action that is measured by the Lagrangian function in the calculus of variations. Recall that all the major laws of physics can be found from the least action principle — Newton's laws, Maxwell's equations, Einstein's gravitational law, etc. Action is the digital currency for all transactions in the Universe, and quantum mechanics describes the processing of action transactions in both linear and rotational Planck currency units, h and $\hbar$. So, surely quantum mechanics should be derivable from the least action principle too? Dirac was keen on this idea, but could not see how to do it.

In the *Feynman Lectures* there is a special lecture devoted to the least action principle (vol II, Chapter 19). Feynman describes how his physics teacher at school, Mr. Bader, noticed that he was looking bored in class one day, and came over to him after class to tell him about the magic of the least action principle. This is a great example of how a teacher can inspire a lifelong passion. Twenty years after the Heisenberg and Schrödinger versions of quantum mechanics, Feynman managed to formulate his own sum over histories version of quantum mechanics using the least action principle. Which way does a particle go? It goes every possible way, but the amplitudes for most paths cancel each other out so that only the true path remains, and that is the one Nature chooses. Unlike normal probabilities which can only range between 0 and 1, or 0% and 100%, amplitudes are complex numbers that can be positive or negative, and so can cancel each other out, in the same way that a wave crest can annihilate a wave trough when they meet. Just as in other versions of quantum mechanics, real probabilities are calculated by squaring the wave function according to Born's rule.

Feynman's version of quantum mechanics is not very useful for calculations, but the mere fact that it has been proved valid provides strong evidence that the underlying operations of the quantum world at small scales are far from meaningless and random, they incorporate a holistic global view as a vital part of the process of seeking perfection. An infinity of hidden variables is hard at work for all of us.

Copenhagen

The 1927 Solvay conference gathered together the greatest minds in physics to discuss the new quantum mechanics, and it proved to be a watershed moment in physics. Einstein and Schrödinger were still holding out for a more realistic and meaningful interpretation of quantum mechanics, but after intense debates, Bohr and Heisenberg's Copenhagen interpretation triumphed and became the standard for 'understanding' quantum mechanics — just understand that it is not understandable. When de Broglie presented his pilot wave theory for guiding the wave function's behaviour, he was harshly criticised.

The Copenhagen interpretation limits what is possible for us to know. It says that at small scales, material objects do not generally have definite properties, and quantum mechanics can only predict the probabilities for things that we can measure. A measurement disturbs the quantum system and makes the wave function 'collapse' into one of the possible, real outcomes. The wave function collapse is considered instantaneous despite happening across a region of space that could include the whole Universe. The instantaneous communication involved in this collapse is just one of the unexplained features of the Copenhagen interpretation. Another is the problem of where to draw the line between observer and observed, measurement apparatus and quantum system. It was to highlight this problem that Schrödinger thought up his well known paradox of a cat that quantum mechanics describes as alive and dead at the same time — until it is observed. Schrödinger had been inspired by a letter from Einstein, in which Einstein had imagined a bomb that, according to the Copenhagen interpretation, must be exploded and not-exploded at the same time.

The Measurement Problem

Up until the advent of quantum mechanics in the 1920s scientists almost unanimously believed that nature was fully ordered, with all processes obeying the laws of cause and effect — this is known as Leibniz's Principle of Sufficient Reason. The triumphant progress of science after Newton seemed only to reinforce the principle of sufficient reason. Use of Newton's mathematical methods allowed Edmond Halley in 1705 to predict the return of the comet that now bears his name in 1758. The appearance of a comet that was formerly attributed to divine or occult powers, but was now shown to be predictable by mere calculation, was a major boost for science. Statistical methods were introduced into physics in the 19th century to deal with the properties of large ensembles of individual entities, such as the molecules of a gas, but this did not undermine the general faith in the underlying order of nature. After all, insurance companies use actuarial data gathered from the whole population of a country to predict the probability of death of an individual person in a particular year, but that does not mean that the life of one person is not subject to the laws of cause and effect. In 1927, with its acceptance of the Copenhagen interpretation of quantum mechanics, physics bowed to the god of randomness.

Quantum mechanics consists of two distinct processes. The first is the Unitary (**U**), or Schrödinger evolution of the wave function, which is fully calculable from the present into the future, but only gives statistical predictions for the possible outcomes of a measurement. The second process is state reduction (**R**), or measurement, in which the quantum state "jumps" to just one of the possible outcomes, and new information has appeared from an unknown source.

In *The Road to Reality*, Roger Penrose uses **U** and **R** to label the pair of processes, and illustrates their time evolution with a diagram like Fig. 11.2, which shows the steady, unitary evolution of the **U** process and the sudden jump, or collapse of the wave function, with each **R** process. Faced with this mysterious and apparently random jumping, you might think that physicists would have treated it as an underlying mystery that needed further investigation, and this is certainly how Schrödinger, Einstein and de Broglie saw it. But Bohr, Heisenberg, Born and others saw things in a very different light; they insisted that there was nothing more to be known, that measurement outcomes are intrinsically random.

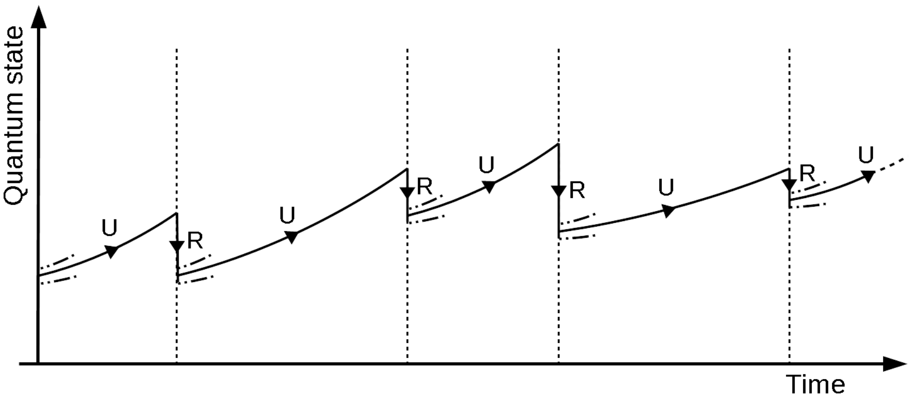


Fig. 11.2 The twin processes of quantum mechanics, **U**nitary evolution (**U**), and state **R**eduction (**R**) by measurement

As long as they are kept isolated from the outer world, quantum systems evolve in time by the **U** process, waves interfering with each other, and all the possibilities represented by the qubits continually having their say. It is this special quantum mechanical **U** process that quantum computers harness to 'compute all possibilities at once', unlike conventional computers which can only consider one thing at a time. The **U** process type of evolution is described as *coherent*, and another question related to the measurement problem is what makes the quantum system *decohere* in the **R** process. After decades of arguments about decoherence, most physicists now accept that contact with the outer world automatically induces decoherence. Some physicists, including Penrose, believe that the choice of outcome in the **R** process is steered by gravity, but we still lack a theory of quantum gravity.

Many Worlds

The many-worlds, or multiverse interpretation of quantum mechanics, asserts that there is no **R** process, the Universe does not make a choice between possibilities, it simply splits into separate Universes for each possibility, and somehow we become separated from all the vast number of alternative Universes required by this theory. Many-worlds has become popular in recent years as it avoids the measurement problem, as well as having other advantages, but it pays a high price in Universes and credibility. If Landauer's principle is true, and all information requires a physical representation, where are these Universes? They seem to be conveniently dispatched into some new, unseen dimensions.

From the perspective taken in this book, we can just regard all these many worlds as hiding within the inner domain of quantum particles, where all possibilities happily coexist until they pass the *reality is truth* test that is required to manifest in the outer world. Recall that it is *within* the fundamental particles that Margolus-Levitin finds most of the processing of the Cosmic Computer. The outer world is then only a coarse approximation to the inner reality, limited in precision by the Planck sized units of communication between inner and outer domains, and the necessity of maintaining equivalence between the dual representations of information. Limited precision is a feature of all digital computers.

> . . . limited precision may also underlie the apparent classical behavior manifested by the events around us. There are many theories that claim to explain that, . . . Some of the explanations may well be correct, and there may not be a need to say more. Nevertheless, a totally quantum mechanical behavior in systems with some complexity and followed for some time, requires a precise evaluation of phases for the competing histories, for the competing Feynman paths. In a world with limited precision relative phases will, eventually, get lost and this will lead to classical behavior. . . the limited precision acts as if the Universe had an unpredictable environment with which it interacts.
> Rolf Landauer, *Information is Inevitably Physical*, in *Feynman and Computation*

Hidden Variables

After 1927, Copenhagen ruled as physicists explored the wealth of possibilities that quantum mechanics had handed them. But Einstein, although he couldn't fault the machinery of quantum mechanics, continued to maintain that it was incomplete, and that there was more to be discovered which might put physics back on the true path of cause and effect, rather than losing itself in the quagmire of randomness. We saw in Chapter 1 that cause and effect is fundamental in eastern thought — there is no room for disconnected randomness. Schrödinger had a lifelong interest in the *Vedanta* philosophy which includes the Upanishads and Bhagavad Gita, and he continued to believe, along with Einstein, that there were more secrets to unlock within quantum mechanics.

Theories that postulate deeper machinery underlying quantum mechanics are called *hidden variable* theories, and de Broglie's 1927 pilot wave theory was the first of these. In 1932 John von Neumann published a mathematical proof that hidden variables theories are impossible, and this was used by the leading proponents of the Copenhagen interpretation to argue that the completeness of quantum mechanics was undeniable. Von Neumann, who later gave us the standard von Neumann architecture for computers, was undoubtedly a genius and one of the greatest mathematicians of his time, so his proof carried weight. Von Neumann's proof was soon shown lacking by the mathematician Grete Herman, but as a woman, and lacking von Neumann's stature, she was ignored. In 1952, David Bohm introduced a holistic quantum potential, a development of de Broglie's earlier pilot wave approach, and showed that a hidden variables theory could work, despite von Neumann's proof. John Bell showed in the 1960s that one of von Neumann's initial assumptions was unjustified, and so his proof was invalid.

Physicists distinguish between two types of hidden variables, local or non-local. A local hidden variable would respect the light-speed communication limit of relativity by only acting at its location in space-time, whereas a non-local one would require some faster-than-light communication channel. This distinction became important in understanding particle entanglement, as we will see.

Spin

The Bohr model of a hydrogen atom consisted of a proton nucleus with an electron orbiting around it, and the possible orbit levels corresponded to fixed multiples of the Planck action unit in its guise as angular momentum. As quantum theory was developed, it became clear that, to account for the fine splitting that was observed in spectral lines, an electron must possess spin around its own axis, as well as orbiting around the nucleus. The energy difference between up or down directions for this inherent spin of an electron was tiny, more than a million times smaller than the energy jumps between orbits in an atom, but it was enough to account for the fine splitting of the spectral lines. The spin axes of electrons and protons point in the same direction as their north-south magnetic poles, and a hydrogen atom has a slightly lower overall energy when the spin of the proton in the nucleus is in alignment with that of the orbiting electron rather than pointing in the opposite direction, hence the split spectral lines.

An electron's inherent spin is quantised, but always takes the fixed value of only half a reduced Planck, $\frac{1}{2}\hbar$, rather than the integer multiples of $\hbar$ attributable to an electron's orbital rotations. Physics assumes that all the electrons in the Universe spin at the same rate, another strong hint that a universal clock is at work. The inherent spin of an electron has only two values, a binary choice of clockwise or anti-clockwise, and these are usually called up and down. You cannot attribute a definite direction in 3D space to the particle's spin axis. Until it is measured, it can be in a superposition of different spin directions — quantum spin seems to operate in a separate spatial domain. All you can do is align your measurement apparatus in your chosen direction of our 3D space, and ask a particle "does your spin axis point in this direction?", and get a yes or no answer. If you repeat the measurement, you will get the same answer again, and if you turn your apparatus through 180 degrees you will find the opposite answer, but you can never know the particle's exact spin direction. With the perspective of this book, we can just explain that an electron's spin can only point inward or outward in the scale dimension which is independent of the 3D directions in space. Simple.

Massless boson particles like the photon carry a full $\hbar$ quota of angular momentum, and gravitons, the postulated messenger particles of gravity, should carry angular momentum $2\hbar$. Another peculiarity of electrons, and other fermions, is that a 360 degree rotation will not return a particle to its original quantum state, a double rotation of 720 degrees is required. This peculiarity is intimately connected with the $\frac{1}{2}\hbar$ value of angular momentum.

The intrinsic spin angular momentum of an electron can be simply added to its orbital angular momentum in an atom to find the total angular momentum that is observed. Nucleus and atom share a common axis through the scale dimension, and communicate with each other through spin↔orbit interactions, yet maintain rotational independence from each other, like one ball rotating freely within another. The magnetic interactions between electron spins in atoms can be harnessed for information processing, this is called *spintronics*, and it is a hot topic for research, as much less energy is needed to flip an electron's spin than to create the current needed to move electrons in a device, so spintronics devices use less power. Bits represented as spin states can also be set more quickly than the bits in conventional computers which depend on charge levels, so computers can be made faster as well as needing less energy.

Entanglement

In 1935, Einstein and his co-workers, Podolski, and Rosen, published one of the most cited papers in the history of physics. The EPR paper was the first description of an effect that Schrödinger later called *entanglement*. According to quantum mechanics, two particles that are separated can share opposite values for a measurable property as the result of a previous interaction between them. This is now routinely demonstrated experimentally by splitting a photon into two lower energy photons that go off in opposite directions, with opposite spins — if one is spin-up, the other is guaranteed to be spin-down. This sounds quite innocent, but when you measure the spin of one of the pair of photons you get a random up-or-down outcome, and if the other photon is observed somewhere far away, how can it know which direction it's spin should take when it is measured? Experiments show that this collusion between entangled particles is robust, and operates much faster than light, and for logical consistency it must be instantaneous. The entanglement link between the particles cannot be used for signalling information from one place to another, because each observer just finds a random outcome for the spin measurement, it is only by means of an ordinary message, limited by the speed of light, that they can find out whether their particle was correlated with the other observer's particle.

The obvious way to explain entanglement is to assume that there are some hidden variables that get set to opposite values when particles interact, and later keep the outcomes of measurements correlated. In 1964 John Bell published a paper called *On the Einstein Podolsky Rosen Paradox* which contained a theorem showing that basic properties of 3D space, along with an experimenter's freedom to orient a spin measurement apparatus at any chosen angle, make it possible to test whether entanglement could be explained by 'local' hidden variables carried within particles, rather than some other hidden variables that depended on non-local effects. Einstein did not believe in non-local, action at a distance effects. He thought that all physical interactions must be local, as any non-local means of communication would break the speed of light limit set by his own theory of special relativity, and one of his motivations for the EPR paper was to show that quantum mechanics was incomplete, because it evidently predicts such nonsensical, non-local effects.

The EPR paper made headlines when it was published in 1935, and caused Bohr to drop everything, and struggle to find a riposte. Bohr did finally manage to fudge together a reply to Einstein's EPR challenge, and philosophers are still trying to unpack his words. He seemed to accept some form of non-local effects while also denying them. Despite Bohr's weak rebuttal of EPR, there were no conceivable experiments in 1935 that could distinguish between the different types of hidden variable, and as entanglement was just one more peculiarity of quantum mechanics, physicists carried on regardless. It was not until thirty years later that Bell's work opened the way for experiments capable of putting non-locality to the test. A string of such experiments, most notably those of Alain Aspect between 1980 and 1982, showed that the quantum world must be inherently non-local. The 100% true record for the predictions of quantum mechanics in experimental tests was undented, and Einstein was shown to be wrong in his belief that the speed of light is a limit for all motion. It may well be the limit for motion of material objects, but it is evidently not for some forms of information.

These conclusions were challenged in 1999:

> All information in quantum systems is, notwithstanding Bell's theorem, localised. Measuring or otherwise interacting with a quantum system S has no effect on distant systems from which S is dynamically isolated, even if they are entangled with S. Using the Heisenberg picture to analyse quantum information processing makes this locality explicit, and reveals that under some circumstances (in particular, in Einstein-Podolsky-Rosen experiments and in quantum teleportation) quantum information is transmitted through 'classical' (i.e. decoherent) information channels.
>
> David Deutsch & Patrick Hayden, *Information Flow in Entangled Quantum Systems*.

Deutsch and Hayden use a model of qubits passing through a network of quantum logic gates, to trace the flow of their entangled quantum information, rather as one might try to keep track of the position of a pea that is hidden under one of the shells in a hustler's shell game. After entanglement, a pair of qubits are separated, and each rotated through a different angle. Because of the entanglement, neither of the qubits could now be measured independently to get information about their rotation angles, this information has become *locally inaccessible*, as if it is encrypted by the entanglement. Nevertheless, this information can, and generally does, spread from one of the entangled qubits to other qubits in its local environment by decoherence, but it is not affected by such interactions. Any of these new qubits can then be sent to meet the second entangled qubit, and used to reveal information about the difference between the original rotation angles of the pair of entangled qubits. Deutsch and Hayden conclude that information shared between entangled qubits does travel inside them, but in an encrypted form that is locally inaccessible.

They also claim that the ability of quantum information to flow through a classical channel, surviving decoherence, is the basis of quantum teleportation, which they also model in their paper.

While Deutsch and Hayden’s argument may be correct for the rather special type of EPR that they discuss, there are simpler versions that do not seem explainable by this analysis. Take for example two experimenters who are separated by a large distance, and who each measure one of the photons coming from a spin-entangled pair created half way between them. Whatever spin is measured by one experimenter for her photon, the other experimenter will find the opposite for his photon. This can be repeated many times, and the results communicated to someone at the midpoint, where the photons were entangled. In this case, nothing passes from one experimenter to the other.

Quantum entanglement is now used in cryptography to make information locally inaccessible to anyone who does not have the right quantum key.

Bohmian Mechanics

Quantum mechanics, even though it remains to this day 'incomplete' in Einstein's view, has been so accurate and useful in its calculated predictions that most physicists have avoided trying to understand it, warned off from this bottomless, career destroying pit — better to 'shut up and calculate'. Nevertheless, some physicists have continued to seek a better understanding of quantum mechanics. David Bohm showed that quantum mechanics is still consistent if a holistic quantum potential is included to guide the evolution of the wave function, taking the role of de Broglie’s pilot wave. Bohm described the world as split into two domains, the *implicate* and *explicate* orders, with a continual process of unfolding and enfolding between the two domains. While Bohm’s description is much the same as the view presented in this book, he only presented a general philosophical picture of his model for the Universe without going into very much detail about its mechanics. This book can be seen as an attempt to update Bohm’s holistic vision of physics for the information age.

> What is primary, independently existent, and universal has to be expressed in terms of the implicate order. So we are suggesting that it is the implicate order that is autonomously active while . . . the explicate order flows out of a law of the implicate order, so that it is secondary, derivative, and appropriate only in certain limited contexts.

> . . . the whole implicate order is present at any moment, in such a way that the entire structure growing out of this implicate order can be described without giving any primary role to time.
>
> . . . a 'particle' is to be understood as a recurrent stable order of unfoldment in which a certain form undergoing regular changes manifests again and again, but so rapidly that it appears to be in continuous existence.
>
> . . . the actual structure, function and activity of thought is in the implicate order.
>
> . . . consciousness can be described in terms of a series of moments. . . each moment is experienced directly in the implicate order. . . through the force of necessity in the overall situation, one moment gives rise to the next, in which content that was previously implicate is now explicate while the previous explicate content has become implicate.
> David Bohm, *Wholeness and the Implicate Order*

The explicate order is our outer world reality, and the implicate order is the subatomic creative source, in which any component of reality contains enfolded within itself the totality of the Universe. The continual process of enfolding and unfolding is like a breathing motion of the Universe. Notice that Bohm places the thinking of the Universe 'the actual structure, function and activity of thought' in his internal, implicate order, just as Margolus-Levitin found the major portion of the Universe's information processing within particles.

Bohm's universal quantum potential guides quantum waves in the same way as de Broglie's pilot waves, and Bohm proved that it is possible to add his potential to the existing quantum theory without changing anything else in the formalism or the results of calculations. The results of the measurement **R** process still appear random to us, but are explained by Bohm as effects caused by the non-local quantum potential, which has a holistic, Universe-spanning view of events.

We have already noted that Dirac found it surprising that quantum particles in a superposition relationship can maintain their phase synchronisation, even when they are moving, and despite the apparent ban on a universal time by special relativity. Bohm showed that this phase synchronisation could be used to show why action has to be quantised — why the Planck exists.

> Let us suppose that the infinity of non-linearly coupled field variables is in reality so organized that in each region of space and time associated with any

> given level of size there is taking place a periodic inner process. The precise nature of this process is not important for our discussion here, as long as it is periodic (e.g. it could be an oscillation or a rotation). This periodic process would determine a kind of inner time for each region of space, and it would therefore effectively constitute a kind of 'inner clock'.
>
> . . . We see, then, that quantisation of action can, at least in this special case, arise out of certain topological conditions, implied by the need for single-valuedness of the clock phases.
>
> . . . the agreement of the phases of all clocks that reach the same point in space and time is equivalent to the quantum condition.
> David Bohm, *Wholeness and the Implicate Order*

We will see in Chapter 13 that the most important 'periodic inner process' of our reality is the inner motion of the electron known as *zitterbewegung*, a dance to the beat of the electron's own inner clock.

It is important to distinguish between phase synchronisation and time synchronisation. If two pendulums of slightly different lengths are set swinging, there will be, every now and then, short time periods when they move in unison — in phase synchronisation. But if we measure time by the total number of swings each pendulum has made, the answers will be different. Phases can agree when clocks disagree, and vice versa.

> . . . we can easily explain the peculiar quantum-mechanical correlations of distant systems by supposing hidden interaction between such systems, carried in the sub-quantum level. With an infinity of fluctuating field variables in this lower level, there are ample motions going on to explain how the correlations are maintained if, while the two systems are still flying apart, we suddenly change the variable that is going to be measured by changing the measuring apparatus for one of the systems. How does the far-away system receive instantaneously a 'signal' showing that a new variable is going to be measured, so that it will respond accordingly?
> David Bohm, *Wholeness and the Implicate Order*

Since the 1960s, when Bohm was writing the above, many experiments have confirmed that quantum theory is correct, and that mysterious, faster-than-light 'signalling' really happens. Bohm pointed out that the liberty we take by extrapolating our current theories unchanged, down to excessively small lengths and times, is at the root of many problems in physics. For example, the proliferation of infinite divergences in field theories has to be removed by the mathematical trick of *renormalisation*, which Richard Feynman described as 'sweeping them under the carpet'.

> . . . the divergences of present-day field theories are directly a result of contributions to the energy, charge, etc., coming from quantum fluctuations associated with infinitely short distances and times. Our point of view permits us to assume that while the total fluctuation is still infinite, the fluctuation per degree of freedom ceases to increase without limit as shorter and shorter times are considered. In this way, field-theoretical calculations could be made to give finite results. Thus, it is clear already that divergences of the current quantum field theory may come from the extrapolation of the basic principles of this theory to excessively short intervals of time and space.
> David Bohm, *Wholeness and the Implicate Order*

By design, Bohm's version of quantum mechanics gave just the same answers as the Copenhagen interpretation, so most physicists have preferred to stick with what they knew, and ignore all the holistic metaphysics that Bohm had introduced, as it gave them nothing new other than balm for philosophic souls. But Bohm thought the implications of our world view go well beyond physics.

> My suggestion is that a proper world view, appropriate for its time, is generally one of the basic factors that is essential for harmony in the individual and in society as a whole.
> David Bohm, *Wholeness and the Implicate Order*

Creating Information

We have already seen how Maxwell attributed his creativity to something beyond himself. This is a common perception of creative people in every branch of human culture, and the transcendental feeling that can accompany creative activity, is a powerful and addictive stimulant that keeps people wanting to create. One of the powerful and liberating experiences in meditation is learning to simply watch thoughts come and go, without attaching to them as 'me' or 'mine'. It is also common for new ideas to appear in different people's heads at around the same time, as if they come from a communal pool of ideas, or what Carl Jung called the collective unconscious.

The source of all 'new' information in the physical world seems to come from the subatomic quantum world. In cosmology, the detailed maps of the cosmic background radiation show the same structure as quantum processes, and the cosmos is considered to be an expanded version of the quantum world. The quantum mechanical **R** process creates new information in the form of actual outcomes of events. When two separate particles interact and go into an entangled state that shares a bit of information between them, a subsequent measurement on one particle reveals the new information of whether it chose to be 0 or 1, and instantaneously, at any distance, a measurement on the second particle will take the opposite value — the newly created information describes which particle is 0, and which is 1.

> If a classical system is in a definite state, . . . , then all the pieces of the system are also in a definite state, . . . If we know the state of the whole, then we also know the state of the pieces. . . But when a quantum system is in a definite state, the pieces of the system need not be in a definite state. In entangled states, we can know the state of a quantum system as a whole but not know the state of the individual pieces. . . the underlying dynamics of quantum systems preserve information, just as the dynamics of classical systems do.
>
> When the pieces of a quantum system become entangled, their entropies increase. Almost any interaction will entangle the pieces of a quantum system. The universe is a quantum system, and almost all of its pieces are entangled. . . . we see that entanglement is responsible for the generation of information in the universe.
> Seth Lloyd, *Programming the Universe*

Lloyd describes the Universe as starting with one bit of information after the big bang, and breeding the rest of its 10^{123} bits from its quantum processes. But this seems to be at odds with both analytical mechanics and quantum mechanics, which maintain that information cannot be created or destroyed. The question of where information comes from is closely related to the question of where everything else in the Universe comes from, and we have seen that Landauer's principle asserts that information and its physical representation are inseparable. The big bang theory postulates an episode of inflation, a faster than light expansion fed by negative energy, that expands the new-born Universe, and primes it with positive energy. But does negative energy exist? And can it carry information? It seems that physics does not yet have a good explanation for the origin of information.

There are many names for the duality of real world and source world. In Yoga philosophy they are often termed the manifest and the unmanifest. Plato envisaged a world of perfect forms as the source of everything in reality. Bohm talks of the explicate and the implicate orders, but also of the measurable and the unmeasurable.

. . . fragmentation originates in essence in the fixing of the insights forming our overall self-world view, which follows on our generally mechanical, routinized and habitual modes of thought about these matters. Because the primary reality goes beyond anything that can be contained in such fixed forms of measure, these insights must eventually cease to be adequate, and will thus give rise to various forms of unclarity or confusion. However, when the whole field of measure is open to original and creative insight, without any fixed limits or barriers, then our overall world views will cease to be rigid, and the whole field of measure will come into harmony, as fragmentation within it comes to an end. But original and creative insight within the whole field of measure is the action of the immeasurable. For when such insight occurs, the source cannot be within ideas already contained in the field of measure but rather has to be in the immeasurable, which contains the essential formative cause of all that happens in the field of measure. The measurable and the immeasurable are then in harmony and indeed one sees that they are but different ways of considering the one and undivided whole.

When such harmony prevails, man can then not only have insight into the meaning of wholeness but, what is much more significant, he can realize the truth of this insight in every phase and aspect of his life.

As Krishnamurti has brought out with great force and clarity, this requires that man gives his full creative energies to the inquiry into the whole field of measure. To do this may perhaps be extremely difficult and arduous, but since everything turns on this, it is surely worthy of the serious attention and utmost consideration of each of us.
David Bohm, *Wholeness and the Implicate Order*

12
The Scale Dimension

Down the Rabbit Hole

> The laws of physics are not symmetrical under change of scale.
> Galileo, quoted in Feynman's *The Character of Physical Law*

> … the fundamental atomic laws, which we call quantum mechanics are quite different from Newton's laws, and are difficult to understand because all our direct experiences are with large-scale objects and small-scale atoms behave like nothing we see on a large scale. So we cannot say, "An atom is just like a planet going around the sun," or anything like that. It is like *nothing* we are familiar with because there is *nothing like it*.
> *The Feynman Lectures* I, 19-3

Alice's experiences down the rabbit hole and, in particular, the importance of changes of scale on our perceptions of the world, seem to have a parallel in the progress of technology in the 20th century. In the normal practice of engineering, as you change one parameter of a design to improve performance, something else usually gets worse, you generally have to make compromises to create something that works. But the pioneers of integrated circuits saw a different reality as they looked down the scale dimension, packing more and more components onto a given piece of silicon. In 1968, Gordon Moore, co-founder of Intel and originator of Moore's law, asked Carver Mead whether quantum tunnelling, the process which permits electrons to spirit themselves through barriers, would be a major limitation on how small the transistors in integrated circuit could be made. Thinking about this, Mead decided to make it the subject of a talk:

> As I prepared for this event, I began to have serious doubts about my sanity. My calculations were telling me that, contrary to all the current lore in the field, we could scale down the technology such that everything got better. The circuits got more complex, they ran faster, and they took less power – WOW! That's a violation of Murphy's law that won't quit! But the more I looked at the problem, the more I was convinced that the result was correct, so I went ahead and gave the talk, to hell with Murphy! That talk provoked considerable debate, and at the time most people didn't believe the result. But by the time the next workshop rolled around, a number of other groups had worked

through the problem for themselves, and we were pretty much all in agreement. The consequences of this result for modern information technology have, of course, been staggering.
Carver Mead, *Feynman and Computation*

Space appears to be the same everywhere, it has the same properties at every point, it is homogenous, uniform, without irregularities. It seems safe to assume that this holds true, whether we consider a small volume or a big volume. But physicists have found that space is not like this, zoom in or out, and it has different properties. Whereas *translation invariance*, guaranteed by conservation of momentum, allows you to move an experiment and get the same results, and *time translation invariance*, guaranteed by energy conservation, allows you to do the experiment at a different time, you cannot necessarily scale your experiment up or down without consequences. The importance of scale differences is apparent in Nature, insects can walk on water, and float in the air, but we cannot. And it is not just physical objects that behave differently at different scales, the properties of space itself change with scale, the smaller the volume of space you want to probe, the more energy you require. Space is not homogenous under changes of scale.

The Lower Limit of Visibility

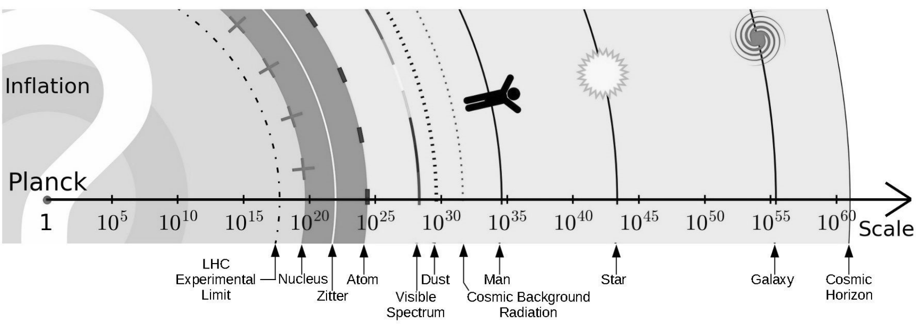


Fig 12.1 A piece of the Atom-Universe-Man mandala showing the span of scales of the Universe.

Visible light, with all its spectrum of colours in which we perceive the world, is just one octave, one doubling of frequency, out of the 202 octaves of the full electro-magnetic spectrum of the Universe. The scale dimension of the Universe spans 61 powers of ten, and $10^{61} \approx 2^{202}$. Fig. 12.1 is a piece of the AUM mandala, shown in the introduction of this book, that illustrates the range of scales. An optical microscope cannot see an atom because light cannot 'see' anything that is smaller than its own wavelength, and an atom's size is about 10^{-10} metre, ten thousand times smaller than the 10^{-6} metre wavelength of visible light. Quantum mechanical wave↔particle duality assigns a wavelength to any mass-carrying particle, and the faster the particle goes, the shorter is its wavelength. An electron microscope, using very fast electrons with smaller wavelengths than light, can see almost down to the atomic scale.

Whether you use light, or electrons, or anything else as your probe below the atomic scale, you need more energy. With ultra-violet light you can see down nearly as far as the atomic scale, but each ultra-violet photon packs much more energy than a visible photon, and can even dislodge an electron completely from its atom, rather than simply changing the electron's orbital energy. The even smaller electro-magnetic wavelengths of x-rays allow you to see atoms, but disturb the atoms more because of their higher photon energies. Whatever electro-magnetic wavelength you probe with, whether it is 10^{-11} metre x-rays, 200 metre radio waves, or any wavelengths in between, the only objects that can emit or absorb the photons are electrons that are bound inside atoms. Gamma rays, with even smaller wavelengths than x-rays, have wavelengths around 10^{-12} metre, midway in scale between atom and nucleus, but they are only produced in radioactive decay and other nuclear processes.

Atoms

Of the particles that make up an atom, a proton is 1836 times heavier than an electron, and a neutron 1839 times heavier. An electron carries a fixed amount of negative charge, and orbits around an atom. A proton carries the same amount of positive charge, and remains in the atom's nucleus. Starting with hydrogen, which has one electron in orbit and one proton in the nucleus, the elements of the periodic table are built up by adding one electron and one proton with each new element: hydrogen has one electron and one proton, helium has two electrons and two protons, lithium has three electrons and three protons, etc. Electrically neutral neutrons are also included in the nuclei of elements above hydrogen, in numbers more or less matching the numbers of protons. Because of the balance of positive and negative charges, every atom is electrically neutral as far as the outer world is concerned, unless it is ionised by losing or gaining an electron.

Because almost all of the mass of an atom is concentrated in its nucleus, and the diameter of the nucleus is 10^5 times smaller than the atom itself, the density of a nucleus is 10^{15} times greater than the density of its surrounding atom. Because the material world is made of atoms linked together by electrical forces, atoms have similar densities to the matter that is made from them. The enormous 10^{15} times increase in density between the atomic scale at 10^{-10} metre, and the nuclear scale at 10^{-15} metre, is the first major change in the nature of space at different scales that I want to highlight.

The second major change is in which forces operate. There are only four fundamental interactions that are known to physics:

1) Gravity
2) Electro-Magnetism
3) Strong
4) Weak

The last two of these, the Strong and the Weak are confined to the atomic nucleus and have no effect at larger scales. Both the electro-magnetic and gravity forces obey inverse square laws, falling off rapidly with distance, but the strong force that holds the protons and neutrons of a nucleus together behaves very differently; above the nuclear scale of 10^{-15} metre it weakens rapidly, and below the nuclear scale it flips from attractive to strongly repulsive — it seems to be designed to maintain the nuclear scale. The weak force is responsible for nuclear decay, and is the only interaction that the elusive neutrinos participate in, and it also does not follow an inverse square law. Our outer world is ruled by the two inverse square following forces of gravity and electro-magnetism. Because gravity is around 10^{40} times weaker than electro-magnetism, and magnetic forces are much weaker than electric forces, we live in an electric world, and as we saw in Chapter 7, the extraordinarily fine balance of positive and negative electric charges in the world leaves us with little appreciation of just how powerful electrical forces really are.

The third major change in stepping down from the atomic scale to the nuclear scale in an atom is the reversal of charge polarity. If you zoom out from the nucleus of an atom in the dimension of scale, charge polarity flips from positive at the nucleus, to negative in the outer electron shells, it is the electrical attraction of these opposite charges that holds an atom together. If you could hover at mid-scale between an atom and its nucleus, you would see positive charge below you in scale and negative charge above you. The Universe is polarised here in the scale dimension — plus in, minus out — and the AUM mandala, or Fig. 12.1, make this evident.

Although physicists have discovered hundreds of particles, it is still only the protons and electrons that:

1. Form matter.
2. Exist for more than a microsecond, 10^{-6} second.
3. Carry mass and the ability to stand still, not always moving at light speed.
4. Carry a quantum of +ve or -ve electric charge.
5. Possess spin and have North/South magnetic poles like tiny bar magnets.

Photon and neutrino particles have unlimited lifetimes, but carry no rest-mass. Neutrons are electrically neutral because they are a sort of marriage of proton and electron. But a neutron's marriage is unstable, and typically ends in a radioactive divorce after about 13 minutes, unless the neutron is locked up inside an atomic nucleus or neutron star. The quark 'particles' that make up the inner constitution of protons and neutrons are not only inseparable from their mother particles, but also morph continually between one type of quark and another. Electrons are found experimentally to have no internal structure, but as we will see in Chapter 13, the Dirac equation, which correctly describes an electron's behaviour even at relativistic speeds, insists that the electron has a four component sub-structure which enables it to perform a mysterious light speed dance called *zitterbewegung* at a scale midway between atom and nucleus. The zitter dance seems to be a key component of the Cosmic Computer's operations, keeping the inner and outer descriptions of the world in synch.

Any picture is ultimately composed of pixels. This is not only true of a picture displayed by a computer, but also of a painting, which is a pattern of pixels made from tiny grains of pigment. If you zoom into a picture to a scale smaller than a single pixel, the picture disappears, and you enter another world — within the pixel. Although atoms are too small for us to see with visible light, they are the pixels of our reality. Within them is a separate reality, the scale range of atoms from 10^{-10} to 10^{-15} metre is a natural boundary zone between two realms of Nature.

At scales in the outer world that are much larger than an atom, Nature's Planck unit of action is so small as to be insignificant, but as you approach the atomic scale, this is no longer true. The situation for the currency of Nature, the Planck, is similar to that for a monetary currency like the dollar. To a billionaire, who often makes transactions involving millions of dollars, the loss or gain of a dollar is insignificant, but to a beggar, it may be the difference between eating and going hungry. If payments and receipts of money are rounded to the nearest dollar, there will be an uncertainty of 99 cents in each transaction. Similarly, the Planck, according to physics, is the fundamental unit of Heisenberg uncertainty in Nature's transactions, and quantum effects grow more important as you descend to the atomic scale:

> To get a feeling of what the Heisenberg relation implies for various objects, we can compare the product of the size of an object times its typical momentum to Planck's constant, h – a measure of how important quantum effects are. For a flying tennis ball, the uncertainties due to quantum theory are only one part in about ten million billion billion billion (10^{-34}). Hence a tennis ball, to a high degree of accuracy, obeys the deterministic rules of classical physics. Even for a bacterium the effects are only about one part in a billion (10^{-9}), and it really doesn't experience the quantum world either. For atoms in a crystal we are getting down to the quantum world, and the uncertainties are one part in a hundred (10^{-2}). Finally, for electrons moving in an atom the quantum uncertainties completely dominate and we have entered the true quantum world governed by the uncertainty relations and quantum mechanics.
> Heinz Pagels, *The Cosmic Code*

Mass

Protons and electrons have lifetimes much longer than the age of the Universe. They carry the fundamental positive and negative electric charges of the Universe, and there are no lighter charged particles that they could decay into. The only way to get rid of one of these particles is to annihilate it with its anti-particle, releasing their combined mass and charge energies in an intense flash of light. But where do you find anti-particles? While we can make them in colliders like the LHC, and observe some arriving from space, everything else we observe seems to be made of ordinary matter. The zitter dance inside an electron incorporates negative mass and the electron's anti-particle, a positron, but these features are hidden from the outer world by the cloak of the uncertainty principle. The fractionally charged quark sub-particles of neutrons and protons, are not able to exist independently, only existing as energy patterns within their mother particles, and continually morphing from one type to another. The name 'particle' seems inappropriate for something that cannot exist 'apart' from something else.

As well as making up the only stable matter that we know of in the Universe, atoms are the only source of light below the frequencies of gamma rays. A free electron, not bound in an atom, cannot emit or absorb photons. If we ignore short-lived particles and light-speed particles like photons and neutrinos, there is not much in space apart from the protons, neutrons and electrons that make up matter, photons of electro-magnetic radiation, neutrinos, and at larger scales, black holes, neutron stars, and other objects. We know very little about the 'dark matter' that is assumed to be present around galaxies to account for the excessive orbital speeds of outlying stars, except that it should have about five times more mass than the normal matter in the Universe. We also know little about the 'dark energy' that has an anti-gravity effect on scales larger than the size of galaxies, and makes up about 75% of all the mass-energy of the Universe. Decades of experimental searches have found none of the particles that theorists have dreamt up to account for dark matter and energy, but there are alternative theories that can explain the missing mass-energy of the Universe by modifying gravitation without the need for more particles. We look at some of these in Chapter 14.

Quantum field theories predict that the vacuum of space at small scales is full of virtual particles popping in and out of existence. But adding up all the energy involved in these processes across the Universe gives a value that is 10^{122} times the measured value. This is sometimes called the 'vacuum catastrophe', or the 'cosmological constant problem'. The evidence from the measured energy density of the Universe actually shows that, where there is no matter, space is almost empty, yet physicists find that the smaller the scale they probe, the more energy they find. One reason for this is that the smaller the scale you want to probe, the more energy you must apply. The content of space at small scales does not just appear for you of its own accord, to see anything, your probe energy must match the energy at that scale, just as a poker player must match an opponent's bet in order to see his cards. This is rather like shining a bright light in a mirror, and having it come back at you.

The positively charged nucleus of an atom is 100,000 times smaller than its surface shell of negatively charged electrons, and there is an enormous electrical attraction between the opposite charges. Why don't the electrons fall into the nucleus? The 'answer' comes from quantum mechanics:

> Why are atoms so big? …What keeps the electrons from simply falling in? This principle: If they were in the nucleus, we would know their position precisely, and the uncertainty principle would then require that they have a very large (but uncertain) momentum, i.e., a very large kinetic energy. With this energy they would break away from the nucleus. They make a compromise: they leave themselves a little room for this uncertainty and then jiggle with a certain amount of minimum motion in accordance with this rule.
> *Feynman Lectures on Physics* I, 2-3.

An electron bound into an atomic orbit travels at about 1% of the speed of light, thousands of times faster than a supersonic aeroplane. The outward centrifugal force from its motion is in balance with the inward electrical attraction to the nucleus. If it gets closer to the nucleus, its position becomes more sharply defined, and the uncertainty principle of quantum mechanics says that it is likely to speed up, increasing its momentum.

Heat

Large objects in frictional contact lose some of their 'big motion' energy down in scale as heat to the 'small motion' of vibrating atoms. But the energy stops at the atomic level, which seem to be a barrier that prevents energy from passing freely up or down in scale. Why does the energy stop at the atomic scale of 10^{-10} metres, rather than leak down to still smaller scales? What stops it carrying on down to the Planck scale of 10^{-35} metre?

The specific heat of a substance is a measure of how much heat energy is required to raise the temperature of a unit mass of the substance by one degree. In *The Principles of Quantum Mechanics*, Paul Dirac discusses the problem of specific heat that had preoccupied physicists greatly before quantum mechanics was formulated:

> Classical statistical mechanics enables one to establish a general connexion between the total number of degrees of freedom of an assembly of vibrating systems and its specific heat. If one assumes all the spectroscopic frequencies of an atom to correspond to different degrees of freedom, one would get a specific heat for any kind of matter very much greater than the observed value. In fact the observed specific heats at ordinary temperatures are given fairly well by a theory that takes into account merely the motion of each atom as a whole and assigns no internal motion to it at all.
>
> This leads us to a new clash between classical statistical mechanics and the results of experiment. There must certainly be some internal motion in an atom to account for its spectrum, but the internal degrees of freedom, for some classically inexplicable reason, do not contribute to the specific heat.
> Paul Dirac, *The Principles of Quantum Mechanics*

Quantum mechanics only permits the photon that is emitted or absorbed when an electron changes its orbit to carry a fixed amount of energy in or out of an atom. An electron in its lowest, ground state orbit cannot emit a photon. But the main barrier to heat flow down in scale is that space just appears to be hotter at smaller scales, and we need our vast accelerators to concentrate enough energy to look down there.

Six Measures of Scale

Fig. 12.2 shows how six different measures vary over the range of scales of the Universe, zooming in or out by a factor of 100,000, 10^5, with each step. This diagram is an alternative way of displaying the scales of the Universe shown in the Atom-Universe-Man mandala. The smallest meaningful scale for physics is the Planck length and Planck time at the left of Fig 12.2. The largest scale is the size of the visible Universe at the right. Length and time are in fixed relation to each other because of the constant speed of light. The bottom three measures are simply different ways of labelling the scale, the fixed speed of light making length and time scales vary in strict proportion with each other. The middle of the range of scales is labelled 'dust', the size of a dust grain weighing in at about one Planck mass. Whereas the Planck length and Planck time are at the smallest scale at the far left, a Planck mass of ordinary matter sits in the middle of the Universe in the scale dimension and no one knows why, this is a major mystery of physics. The dust scale sits between the visible light spectrum and the CBR, cosmic background radiation, in the central Goldilocks scale zone of the Universe.

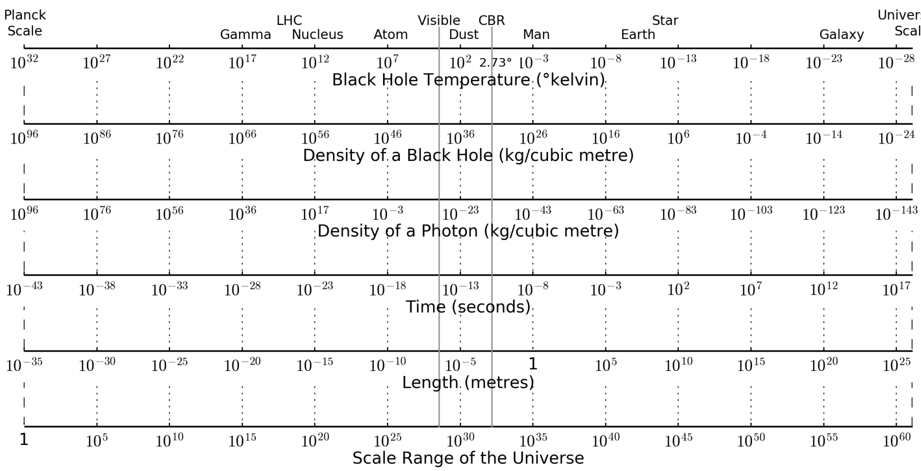


Fig. 12.2 Six measures over the range of scales of the Universe.

The *density of a photon* measure, shows how much energy a photon packs into a volume of space of diameter equal to its wavelength. This measure changes faster than any of the others, going as the fourth power of scale. It shows that if a photon could exist at the Planck scale, it would have an energy density of 10^{96} kg/m^3, whereas a photon with a wavelength spanning the Universe, would have an energy density of only 10^{-147} kg/m^3, a 10^{243} difference. (Note that mass units are used here for convenience, despite the fact that photons have no rest-mass — we take advantage of the $E=mc^2$ equivalence of mass and energy).

The density of ordinary matter made of atoms is about 10^3 kg/m^3, and Fig. 12.2 shows that the energy density of an x-ray photon, of wavelength equal to the width of an atom, is 10^{-3} kg/m^3. This means that the photon's energy is still a million times weaker than the rest-mass energy of an atom occupying the same volume. One step to the left from an atom in Fig. 12.2, at the nuclear scale, a gamma-ray photon's energy density is 10^{17} kg/m^3, 10^{14} times greater than normal matter, and close to the rest-mass energy density of the nucleus itself. This is enough energy to create new particles in the form of matter and anti-matter pairs, whose effects can be calculated in quantum field theory, but which also give rise to the infinite answers that must be spirited away by the mathematical trick of renormalisation.

Going further to the left in Fig. 12.2, below the nuclear scale, there is no stable matter. The highest energy natural gamma-ray photon so far observed had a density of about 10^{36} kg/m^3, and this is marked as Gamma at the top of Fig. 12.2. LHC marks the smallest scale of 10^{-17} metre that we can probe with the Large Hadron Collider, using the energy released by head on collisions of protons travelling at close to the speed of light. The LHC accelerates protons in an evacuated tube, 27 kilometres in circumference, using superconducting magnets to keep them on their circular course. To probe down to the Planck scale, it has been estimated that an accelerator the size of a galaxy would be required. As Fig. 12.2 shows, the LHC achieves energies only about 1000 times less than the Gamma particle, the most energetic cosmic ray particle ever observed.

Black Holes

A black hole's mass is directly proportional to its radius, rather than the cube of radius that applies to ordinary matter. Fig. 12.2 shows that a black hole's density changes by a factor of 10^{122} over the 10^{61} scale range of the Universe, varying in proportion to the scale squared. At the Planck scale, energy would be so densely packed that general relativity predicts that its self-gravity would make it form a black hole. Yet perversely, the black hole temperature measure in Fig. 12.2 shows that the Planck temperature is 10^{32} Kelvin, making it hotter and brighter than anything else in the Universe — the fires of hell? At the other extreme, the Universe as a whole has a very low observed energy density which is close to what general relativity predicts for a black hole of that size — so is the Universe itself a black hole? Physicists do not know, but it must at least be close to being one. It is strange that a large, low density, object could have enough self-gravity to form a black hole, but this is due to the fact that a black hole's mass is not proportional to its 3D volume like any other object, but to its 1D radius, a scale dimension measure. If you increase the mass of a black hole, the radius goes up by the same factor, but the volume goes up much faster, and the density, which is mass divided by volume, goes down. This is why the extremely large black holes that seem to be at the core of some galaxies are much less dense than ordinary matter.

Black holes are normally assumed to be very black, and very cold, and this is true of large black holes like the super-massive one at the centre of our Milky Way galaxy. But, as you can see from the black hole temperature measure at the top of Fig. 12.2, the smaller the black hole, the hotter it should be. Stephen Hawking calculated that black holes must radiate, and his formula says that a black hole's temperature is inversely proportional to its radius or mass, and the wavelength of the emitted radiation is equal to the black hole's diameter. So we can see from Fig. 12.2 that a black hole with a diameter of 2 millimetres, would have the same temperature as the CBR, the cosmic background radiation that fills space, and is taken to be a cooled relic of the big bang origin of the Universe. The coldest place you could find in outer space would be far from a star, sitting in the bath of the 2.7°K CBR radiation. Anything warmer than this would tend to lose heat energy by radiation, and anything cooler would tend to gain heat energy from the CBR. Whereas a black hole larger than 2mm would be colder than space, growing its mass by the nett gain of energy, a black hole smaller than 2mm would be hotter than the CBR, and radiate its energy and mass away, getting hotter as it grew smaller, until theory predicts it would disappear in a great flash. Although theory predicts that the big bang should have created suitable small black holes, no death flashes of such holes have been observed.

The Holographic Principle

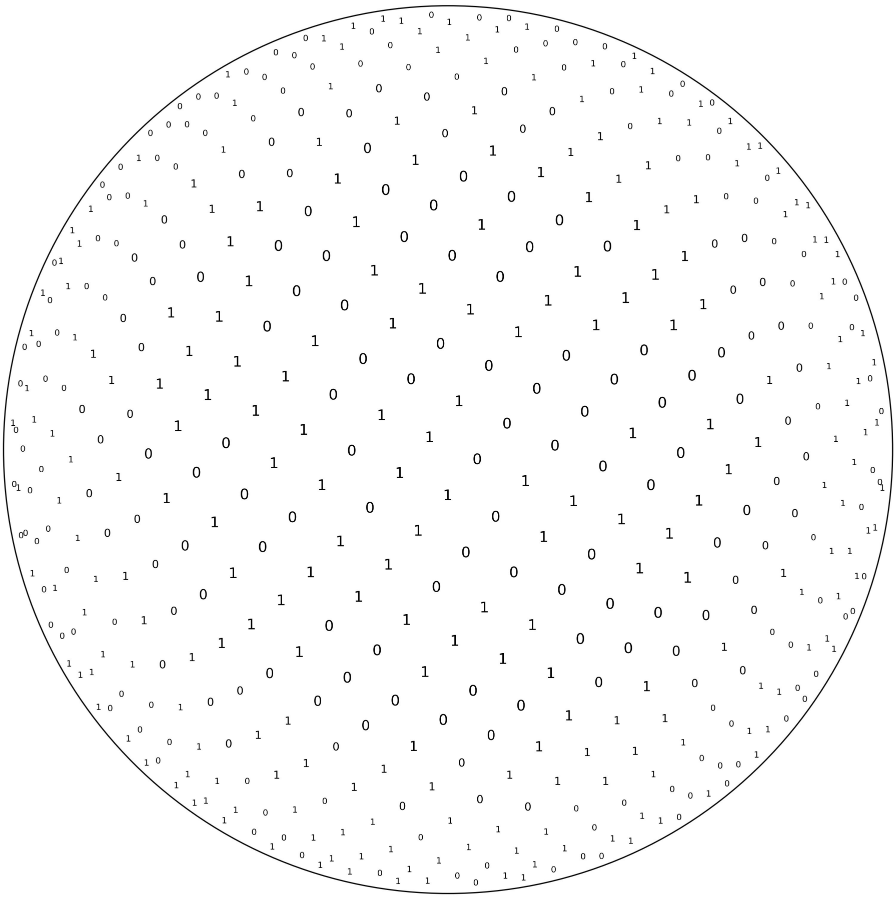


Fig. 12.3 The Holographic Principle
All the information in a spherical volume of space is 'written' on its surface.

Hawking's model of black hole radiation also predicted that black holes must possess entropy, or information content, proportional to their surface area. All the information describing everything the black hole has devoured is written on its surface with one bit occupying four Planck areas of 10^{-70} sq. metre. It was later realised that because you cannot put too much mass-energy in any region of space without it collapsing into a black hole, the information content of not only black holes, but any region of space, is limited not by its 3D volume, but by its 2D surface area. It seems that Nature projects an alternative 2D representation of all the information describing the contents of a 3D region of space onto the region's boundary, and this is illustrated in Fig. 12.3.

Holograms have a similar ability to represent a 3D object on a 2D surface. A hologram is created using coherent laser light of fixed wavelength that can carry extra depth information in interference patterns. The *Holographic Principle* was proposed in 1993 by Nobel Laureate Gerard 't Hooft, and elaborated by Leonard Susskind. It asserts that the Universe is like a hologram, our apparently 3D universe having a dual description of its contents 'painted' on a distant 2D boundary surface. The holographic principle is now widely accepted, and has been used by Erik Verlinde in his theory of *Entropic Gravity*, which describes gravity as a force resulting from the statistical tendency of entropy to increase. This theory seems to account for dark matter and dark energy better than general relativity — see Chapter 14.

As we saw in the Introduction, Seth Lloyd has calculated that the Cosmic Computer has so far performed 10^{122} operations on 10^{92} bits, and continues to process data at 10^{105} operations per second. The bit count comes from thermodynamic arguments which assign bits to elementary particles. But Lloyd also suggests that the bit count might reach 10^{122} if the bits stored by quantum gravity (which we cannot yet describe) are taken into account. If Lloyd is correct, the Universe may be projecting the result of every logical operation in its history on its 2D boundary surface, like that represented in Fig. 12.3.

As gravity is responsible for collapsing matter and energy into black holes, it also seems to play the key role in the holographic projection of information through the scale dimension, threading up through every particle from small to large.

> Oh Arjuna! There is nothing higher than Me; all is strung upon Me as rows of pearls upon a thread.
> Bhagavad Gita VII

If there is an outer 2D holographic boundary at the largest scale, there should equally be a common inner focal point for all the Universe's information at the smallest scale. It is particles that form the hardware of the Cosmic Computer and must also bear the responsibility for projecting their information out to the boundary. It seems reasonable to suggest that they also project it inwards, downwards in scale in compressed, superposed form, towards a common centre at the Planck scale, filtering it through this cosmic viewpoint of all atoms in creation, before sending it outwards again to its separated form on the cosmic boundary.

Large Numbers

There is a factor of 10^{20} difference in scale between a proton and the Planck scale, and a 10^{40} difference between proton and the size of the Universe. There are also around 10^{80} particles in the Universe. Many physicists, including Eddington and Dirac, have believed the remarkable relationship of these large numbers is no coincidence and sought deeper explanations.

As the radius of a black hole is directly proportional to its mass, we can calculate the size that a black hole of the proton's mass would have. This is called a proton's gravitational radius and is 10^{20} times smaller than the Planck scale, and so 10^{40} times smaller than the proton, just the same factor that the proton is smaller than the Universe. The factor of 10^{20} also appears as the ratio of the Planck mass to the proton mass. Extending the Universe down by a factor of 10^{20} below the Planck scale to a proton's gravitational radius, would not only leave the proton sitting in the middle of the Universe's range of scale, but also make the overall range of scales about 10^{80}, approximately equal to the number of atoms in the Universe. You can see why physicists have found these large numbers so tantalizing.

Fundamental Clocks

In his 2001 paper *From computation to black holes and space-time foam*, Y Jack Ng derives fundamental limits for any physical clock, using only the uncertainty principle from quantum mechanics, and a black hole restraint provided by general relativity. (These are the same restraints that are illustrated as the forbidden areas of Majid's diagram in Fig. 5.3). Quantum mechanics says that more mass gives a clock shorter, more precise, ticks, as well as greater running time without losing accuracy. On the other hand, general relativity says that too much mass will make the clock collapse into a black hole. Assuming also that a clock's time for one tick cannot be less than the time light would take to travel across it, Ng balances the conflicting mass requirements of QM and GR for his clock, to find that the ratio of a clock's running duration time to its tick time must be less than the square of the ratio of its tick time to the Planck time.

$$T/t \leq (t/t_p)^2$$

T = total running time, t = clock tick time, t_p = Planck time

The left side of this equation counts the clock's total number of ticks in the outer world T/t. The right hand side is the square of the count of inner Planck time ticks in one of the clock's ticks — beautifully mysterious! Ng does not discuss actual particle clocks in his papers, but if we put the numbers for a proton as a clock into Ng's formula, running with its natural tick time of 10^{-23} second for the 10^{17} seconds age of the Universe so far, we get $10^{40} \leq 10^{40}$, which shows that a proton is as small as it could be in the scale dimension while still being able to keep its ticking accurate over the age of the Universe. The mysterious relationship between the 'inner' and 'outer' time of a clock applies equally well to length scales thanks to the constant speed of light.

Why is this internal time measure squared? Nobody knows, but in his derivation Ng follows Wigner in modelling the spread of the clock's quantum wave packet as a random walk, and this results in the spreading of the wave packet being proportional to the square root of the number of time steps. Ng's formula suggests that time outside the proton, which runs from the t of a proton tick, up to the age of the Universe T, is linked to the product of two inner times that each run from the Planck time t_p, up to the proton tick time t. Perhaps there is a deep connection between this squaring of inner time and the squaring of the wave function that quantum mechanics needs to give a real world outcome probability.

In Chapter 13, we will be solely concerned with the inner behaviour of the electrons that rule the outer world, but remember that it is protons that provides the more accurate clocks for stabilising atoms, as they carry 1836 times the rest-mass of electrons. While the middle of the logarithmic scale dimension is the square root of its total span $\sqrt{10^{61}} \approx 10^{30}$ Planck lengths, it seems that the Universe has a sweet spot for clocks at the cubed root of its overall span in the scale dimension, $\sqrt[3]{10^{61}} \approx \sqrt{10^{20}}$ Planck lengths, and that is just the nuclear scale where the protons at the heart of every atom sit.

13
The Zitter Dance

Electrons Rule the World

Along with the massless photon of light, the fundamental particle that most directly affects our lives is the electron. As we have already seen, it is only the trio of neutron, proton and electron, that contain rest-mass and exist for more than a microsecond, and they are also the component particles of atoms. Whereas neutrons and protons contain quarks, sub-particles that cannot exist outside their hosts, experiments have not found any such internal structure in electrons, so as well as being the first sub-atomic particles discovered by physics, electrons seem to be truly fundamental. In an atom, the protons and neutrons are confined in the nucleus, and it is only electrons that face out into the world above the atomic scale, generating all the electromagnetic effects and photons that keep our reality running, while maintaining an energy intense electric and magnetic dialogue with the nucleus. If the Universe does maintain dual internal and external representations of its information, electrons must be the principal routers that carry information between the two domains, while dancing around the oppositely charged protons of the nucleus.

The only other force that operates at scales greater than an atom is gravity, which being 10^{40} or so times weaker than electro-magnetism, is negligible unless something with at least the mass of an asteroid influences your calculations. The world inside the atomic nucleus has its own distinct new forces, the weak and the strong, but these are restricted to the nucleus, which we will not be concerned with.

The Hydrogen Atom

For simplicity we will ignore the heavier atoms and focus on the hydrogen atom, which consists of just a single negatively charged electron orbiting a single positively charged proton nucleus. Classical (i.e. pre-quantum) physics predicts that the intense electrical attraction between the particles will draw the electron right down to the much heavier proton at the atom's nucleus, with the electron radiating light as it goes. But we know that atoms are stable, so this does not happen. Quantum mechanics comes to the rescue with the uncertainty principle, which makes a particle's momentum increase if the bubble of space it is allowed to occupy gets smaller. To an electron confined within an atom, this means that the closer it gets to the proton, the more it suffers from quantum claustrophobia, and the faster it tries to flee. More momentum also means more kinetic energy, and in the Feynman Lectures I, 38-4, Feynman estimates the size of a hydrogen atom by finding the radius where the sum of the electron's orbital kinetic energy and electrical potential energy is as low as possible. This minimum occurs where the rate of change of the total energy with radius is zero, which is also where the force due to the potential energy of electrical attraction is exactly balanced by the centrifugal force due to the kinetic energy of the electron's motion.

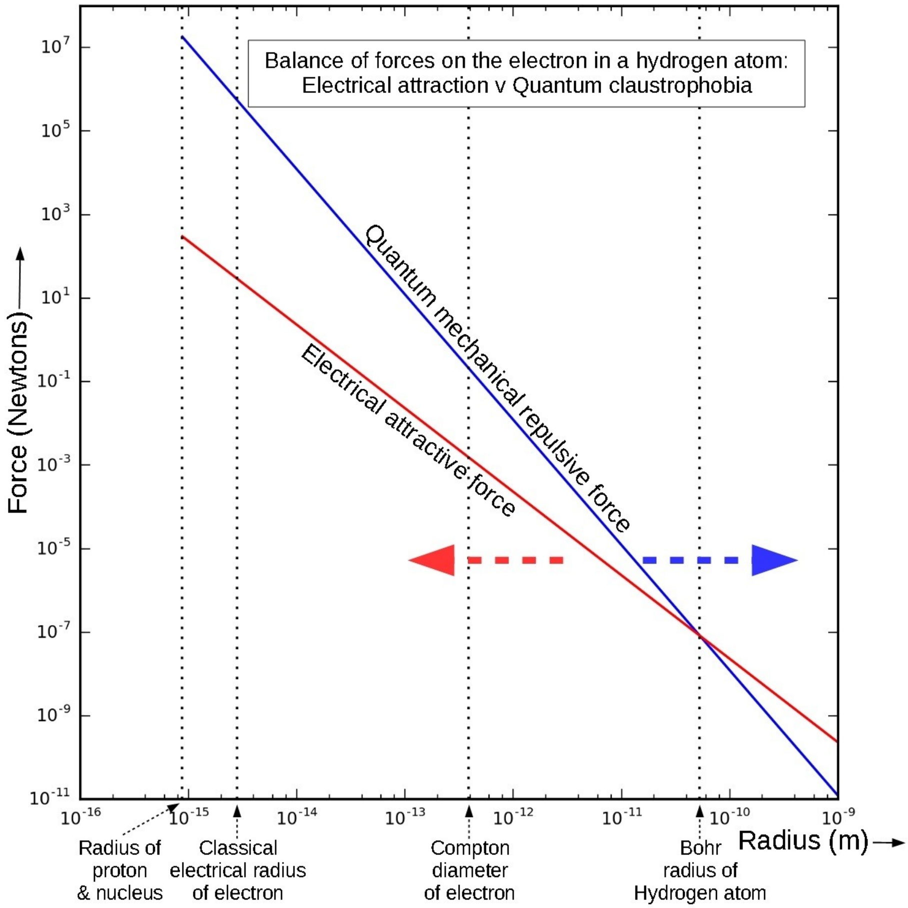


Fig. 13.1 An electron in a hydrogen atom is held by a balance of forces.

Fig. 13.1 ranges over the span of scales between atom and nucleus, and shows the balance of radial forces experienced by the electron in a hydrogen atom as it swirls around the proton nucleus. The two diagonal lines show how the opposing forces of electrical attraction and centrifugal (or quantum mechanical) repulsion vary with distance from the nucleus and balance each other out at the Bohr radius of a hydrogen atom. Although the electron has some probability amplitude to be anywhere around the proton, the Bohr radius is its most probable location, and this makes it the effective radius of a hydrogen atom. Atoms of other elements have similar sizes of around 10^{-10} metre.

We can make use of the dimensional analysis that was described in Chapter 5 to estimate the electron's orbital speed at the Bohr radius of an atom. If we divide the reduced Planck $\hbar$, which has dimensions ML^2/T, by the Bohr radius which has dimension L, we get the electron's momentum with dimensions ML/T. Further dividing by the electron's mass M, we get its speed L/T. Putting in the measured values for the Planck, the electron's mass, and the length of its Bohr radius, we find the electron's speed to be about 1/137 times the speed of light. The pure dimensionless number 1/137.035999084… is called the fine structure constant, α, and it is one of the most deeply mysterious, but accurately measured quantities in physics — Feynman recommended that physicists should have this number displayed on their office walls to remind them how little they know.

The Compton Wavelength

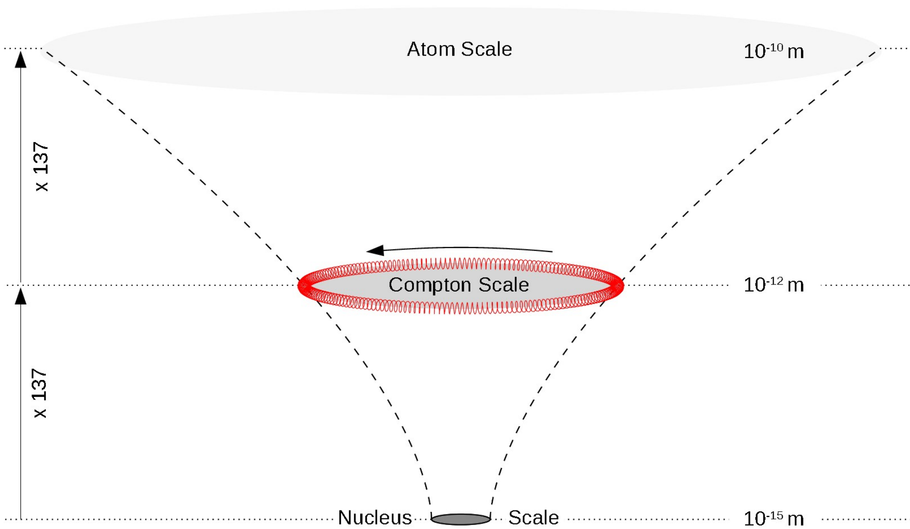


Fig. 13.2 Three important scales for an electron in a hydrogen atom.

Top: Bohr radius, the outer radius of the atom.

Middle: Compton scale of electron, the zitter scale.

Bottom: Classical electrical radius of electron ≈ nuclear scale.

In 1924, three years before Heisenberg proposed the uncertainty principle, Louis de Broglie postulated that, just as light has both wave-like and particle-like properties, mass-carrying particles like electrons also have wave-like properties, with the wavelength proportional to a Planck divided by the particle's momentum: $\lambda=h/mv$, where λ = wavelength, h = 1 Planck, m = mass and v = velocity. More momentum means smaller de Broglie wavelength. The uncertainty principle ensures that any particle with rest-mass has a confinement scale where it is forced to travel at light speed c, so that $\lambda=h/mc$; this scale is called the particle's Compton wavelength. If you imagine one wavelength of the electron wrapping around a circle, the circle's radius is called the reduced Compton wavelength, or Compton radius of the particle. In Figs. 13.1 & 13.2, an electron's Compton diameter (double its Compton radius) is shown, and it lies a factor of α down in scale from the hydrogen atom's Bohr radius, and also a factor of $1/\alpha$ up from the next important scale for an electron, its electrical radius, which is close to the scale of the nucleus.

The electrical radius of a particle is found using pre-quantum, classical physics, by calculating the scale at which confinement of the particle's charge, would increase its self-repulsive electrical energy enough to account for all its $E=mc^2$ rest-mass energy. The electron's classical electrical radius is quite close to the proton radius scale of a hydrogen atom's nucleus, and this is also the scale where the strong and weak forces operate, and virtual electrons and positrons start to arise from the vacuum according to quantum field theories.

In 1922, Arthur Compton found that x-rays that are scattered by atoms lose a little of their energy, decreasing their frequency, and increasing their wavelength. He could account for the phenomenon with classical physics by modelling the collision of x-ray photon and electron as if they were two billiard balls colliding. He found that if the photon was deflected at 90 degrees to its approach, its wavelength λ increased by exactly one Compton wavelength of the electron, h/mc, the increase for any other deflection angle θ being given by the formula: $\lambda in – \lambda out = (h/mc)(1-\cos\theta)$. This 'Compton effect' was a strong piece of evidence for the wave↔particle duality of matter that de Broglie proposed in 1924.

The Compton wavelength of the electron is a crucial scale in physics for many reasons. For example, we saw in Majid's diagram of Fig. 5.3 that the electron and proton lie on the photons line that marks the quantum limit for any mass-carrying entity in the Universe, they would have to gain mass to get any smaller. An electron's $E= mc^2$ rest-mass energy is the same as the energy carried by a photon of wavelength equal to the electron's Compton wavelength. To observe an electron at its Compton scale you would have to bombard it with an x-ray photon of similar wavelength, and the disturbance is enough to eject the electron from its atom by the process called Compton scattering.

The Ring Model of the Electron

Physicists normally consider the electron as a particle with a point-like origin of zero radius, and this is justified by the inability of experiments to detect any internal structures in electrons like the quark sub-structures found in protons and neutrons. But when an equation requires you to divide by a radius of zero, you get nonsense like the predicted infinite energy of a point particle, and how could something with zero radius have angular momentum? These questions are examples of the widespread singularity problem in physics that was mentioned in Chapter 1.

After its discovery by J.J. Thomson in 1897, an electron was first modelled as a unit of charge spread over the surface of a sphere, the finite size of the sphere avoiding the singularity problem. But what would stop the sphere exploding from the self-repulsion of its charge? And if the radius of the sphere was the classical electrical radius, which could already be calculated, its surface would have to travel much faster than light to produce the electron's magnetic moment.

In 1915, Alfred Parson suggested that an electron behaves like a ring of negative electricity spinning very fast about its axis, and that chemical bonds result from two atoms sharing a pair of electrons. In the ring model shown in Fig. 13.3, the electron's charge travels endlessly around a Compton diameter ring at the speed of light, generating a magnetic field that threads through the ring.

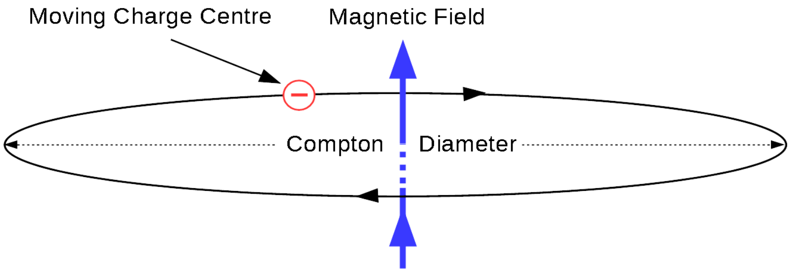


Fig. 13.3 The Ring Model of an electron.
The electron's charge circulates around its centre of mass in a ring of circumference equal to its wavelength, generating its magnetic field.

Although the ring model was shown to be inadequate when the Schrödinger equation was discovered in 1926, it has been revisited many times by physicists, and developed into physical models that can more accurately account for the electron's behaviour. It is remarkable that this simple and visualisable model can predict so much, and later improvements to the model can do much more, as we will see. As we found in Chapter 7, it was Maxwell's slightly unphysical model of magnetic vortices and idle wheels that allowed him to write down his equations, but most of physics today is forbiddingly abstract and mathematical — is it possible to make physics more understandable?

The Schrödinger Equation

The Schrödinger equation is the most useful tool of quantum mechanics and is quite accurate in most situations, including when studying an electron's orbital motion in a hydrogen atom at less than 1% of the speed of light. But for highly accelerated electrons, and other particles that approach the speed of light, relativistic effects become important, and the Schrödinger equation is no longer valid. Schrödinger had originally tried to find a wave equation that was relativistically correct, and he first tried what is now called the Klein-Gordon equation, but found it did not give results that agreed with experiments. It was only after abandoning conformity with relativity that he found his famous equation, which is written on his grave in the simple form:

$$i\hbar\dot{\Psi} = H\Psi \qquad \textbf{The Schrödinger Equation}$$

where i is the imaginary unit, $\hbar$ is a reduced Planck, and Ψ (psi) is the wave function. The dot on top of the wave function Ψ on the left means Ψ's rate of change with time, and H is the total energy of the system, which physicists call the Hamiltonian. So the equation says that the rate of change of the wave function Ψ in imaginary Planck units is equal to the total energy acting on Ψ in real units. The wave function Ψ uses complex numbers which, as we saw in Chapter 4, contain two compartments, real counting in units of 1, and imaginary counting in units of i, that are bound together into a composite 'complex' number whole. We also saw that squaring a complex number by multiplying it by its complex conjugate (a mirror reflection in the imaginary i dimension which flips the + or – sign of its imaginary part) yields a purely real number. The complex conjugate of the wave function Ψ is written Ψ^*, and it is the square of the wave function written $\Psi\Psi^*$ that is interpreted as giving the probability of a measurement outcome. According to quantum mechanics, an electron in an atom does not have a definite position but is smeared in a cloud of probability, with ΨΨ* describing its likelihood to be found at any given point. Ψ and Ψ* each carry *all* the information in the wave function, they are yet another pair of dual quantities.

We saw in the A.C. power example of Chapter 4 that the imaginary part of a complex number is important in engineering to describe the out-of-phase components of a wave, and that phase effects have to be taken into consideration to prevent A.C. electrical machines from destroying themselves. The phase of a quantum wave function is not measurable in the outer world, it hides inside the Planck unit, and the procedure of squaring the wave function dispenses with it. Although phase plays a crucial role in the inner workings of quantum mechanics, allowing waves that are in phase to reinforce each other while out of phase waves cancel each other, the standard Copenhagen interpretation of quantum mechanics assumes that phase has no significance in the outer world, despite the fact that it is precisely the extra powers offered by phase relationships that make quantum computers possible.

The Wave Equation

If you unpack the Hamiltonian function H for the total system energy which appears on the right hand side of Schrödinger's equation written above, it hides a double derivative, a rate of change of a rate of change, in spatial dimensions, whereas there is only a single derivative with time in the dotted Ψ on the left hand side. A single derivative is called first order, and a double derivative is called second order. A typical wave equation has second order differentials on either side, as shown by the small superscript ^{2}s that you can see in the wave equation below, which describes the sideways displacement y of a plucked string with mass per unit length μ, string tension T, and x measuring along the string:

$$\frac{\mu}{T}\frac{\partial^2 y}{\partial t^2}=\frac{\partial^2 y}{\partial x^2} \qquad \textbf{The Wave Equation}$$

All solutions to the wave equation are superpositions of waves travelling in opposite directions, and the speed of the waves is the square root of the ratio of the string's density to its tension μ/T. As we saw in Chapter 7, Maxwell manipulated his fundamental equations of electro-magnetism to find a wave equation that turned out to describe the propagation of light. Just as the square of the speed of waves in a string depends on the balance between the string's mass-inertia and tension, Maxwell found that the square of the speed of light c through any medium depends inversely on the product of the electric and magnetic constants, μ and ε, of the medium like this:

$$c^2 = \frac{1}{\mu\,\varepsilon}$$

If we imagine, along with Maxwell, that space is filled with the magnetic cells and electrical idle wheels of his spinning cells model that we looked at in Chapter 7, the magnetic constant describes the inertial resistance to change of rotation speed of the cells, and the electric constant describes the elasticity of the cells.

The Dirac Equation

The Klein-Gordon equation that Schrödinger had found inaccurate in its description of electrons, is a wave equation of second order on both sides, and is relativistically correct, and so accurate for high speed particles. Although not valid for mass-carrying fermion particles like electrons, protons, and neutrons which each have only ½$\hbar$ spin, it has turned out to work for massless boson particles like photons which each carry ħ unit of spin. In 1928, Dirac managed to factorise the Klein-Gordon equation by taking a sort of square root of it, and concoct a new equation that was first order on both sides, relativistically correct, and able to correctly account for the behaviour of fermions like electrons. Dirac did this by using enlarged 4 x 4 matrices of complex numbers that incorporated the 2x2 Pauli spin matrices described in Chapter 4. If you rotate an object in 3D space about one axis, and then rotate it again around a different axis, the order in which you do these rotations affects the final orientation of the object. Rotations in 3D space behave in the same non-commutative, 'order matters', way as matrix multiplication.

On his memorial stone in Westminster Abbey, Dirac's equation is written like this:

$$i\,\gamma\cdot\partial\,\Psi = m\,\Psi \qquad \textbf{The Dirac Equation}$$

The ∂ symbol here means take the differential of Ψ, which is Ψ's rate of change compared to something else. This is similar to the Schrödinger equation above, where the dotted Ψ means the rate of change of Ψ with time. The *m* in the Dirac equation is the mass of the particle it describes, and this is equivalent to the Hamiltonian H, or total energy, in the Schrödinger equation. The fundamentally new components in Dirac's equation are his 4x4 γ spin matrices which multiply ∂Ψ in a dot product. Note that the simple form shown above hides the complexities of the many sub-components of Ψ and γ.

Dirac's equation was considered quite miraculous, correctly describing not just an electron, but any fermion, or particle of matter. But it came at the price of splitting Schrödinger's single wave function Ψ into four component parts Ψ_1, Ψ_2, Ψ_3, Ψ_4, which allowed for two possible spin directions, usually called up and down, and two versions of electric charge, positive as well as negative. Dirac interpreted the positively charged component of the wave function as an anti-electron, or positron.

Dirac gave another reason for Ψ having four components:

> … the spin of the electron requires the wave function to have two components. The fact that the present theory gives four is due to our wave equation having twice as many solutions as it ought to have, half of them corresponding to negative energy.
> P. A. M. Dirac, *The Principles of Quantum Mechanics*, 4th edition, 1957

Negative energy appears quite often in physics, and the sea level of zero energy between positive and negative can usually be set at will — this is another kind of symmetry freedom. All the kinetic energy of moving matter and its internal $E = mc^2$ energy is considered positive. The energy stored in a gravitational field is taken as negative. The energy stored in the Earth's gravitational field by an infinitely distant mass is taken as zero, and goes down, getting more and more negative, the closer the mass is to Earth. Several physicists, including Stephen Hawking, have suggested that the Universe has zero total energy, the negative gravitational energy exactly balancing the positive energy of matter and radiation. Recall also that in Chapter 6 we saw that the principle of least action, which offers a way to derive all the most important equations of physics, is enforced by making the difference between kinetic energy and potential energy as small as possible.

If there are negative energy states, the questions arise, why don't we observe any negative energy states?, and why doesn't all the positive energy just flow down into the negative energy? An electron's anti-particle, a positron, still has positive energy. The states of negative energy in Dirac's equation are inseparable from the positive energy states, they are two components bound together as part of a quartet. Enrico Fermi proved that without the contribution from negative energy states, electrons would not be able to produce the Thomson scattering of photons that is observed in experiments, so they must be at work, whether we like it or not.

Dirac managed to solve his negative energy problem with some creative thinking. He postulated that the Universe is filled with negative energy states of electrons that are already full, so there is no room for positive energy to flow down into them. These negative energy states filling the Universe are now called the *Dirac sea*. A 'hole' in this negative energy sea would appear as a positron with positive energy. Positrons were observed by Carl Anderson in 1932, betrayed by the tracks they left in a cloud chamber exposed to cosmic rays, their positive charge causing them to turn in the opposite direction to negatively charged electrons when passing through a magnetic field.

When Dirac solved his equation to find the instantaneous velocity of a free electron, he found it to be plus or minus the speed of light, ±c, even if the electron is standing still!

> Since electrons are observed in practice to have velocities considerably less than that of light, it would seem that we have here a contradiction with experiment. The contradiction is not real, though, since the theoretical velocity in the above conclusion is the velocity at one instant of time while observed velocities are always average velocities through appreciable time intervals. We shall find upon further examination of the equations of motion that the velocity is not at all constant, but oscillates rapidly about a mean value which agrees with the observed value.
> P. A. M. Dirac, *The Principles of Quantum Mechanics*, 4th edition, 1957

By extending his equation to describe how an electron interacts with an external magnetic field, Dirac was able to show that the electron has an inherent spin, and a magnetic moment like that of a bar magnet with North and South poles. This spin gives it an angular momentum of ½ħ around an axis aligned with its magnetic poles. In an atom, an electron's inherent spin angular momentum is coupled to its orbital angular momentum around the nucleus. Dirac found that the orbital angular momentum is not constant, but fluctuates, only the sum of the two forms of angular momentum being constant. There is a coupling, a sharing of rotational motion and energy between an atomic electron's inherent spin and its orbital spin, and the fixed size of a Planck makes it impossible to measure them separately. This rotational coupling of an electron in an atom may be a vital part of a communication channel in the scale dimension that could account for phenomena like the instantaneous correlations of entanglement, and the in↔out duality of information that is postulated in this book.

Ever since its discovery in 1928, physicists have been fascinated by the Dirac equation, it integrates spin into the electron's equation of motion, it reveals the existence of anti-matter, and hides new features that are still being discovered 90 years later — it remains a treasure chest full of secrets. It also became the main foundation of quantum electrodynamics, QED, the quantum field theory that quantises fields as well as particles with a procedure called second quantisation. While Richard Feynman won a Nobel prize for his work on QED, and liked to say that it was the most accurate theory known to physics, his hero Paul Dirac never liked QED much, preferring other ideas that did not have to rely on the dubious renormalisation procedure of cancelling one infinity with another.

Zig-Zag Particles

The four components Ψ_1, Ψ_2, Ψ_3, Ψ_4 of the Dirac wave function are called spinors, mathematical entities discovered in the early 1920s by Elie Cartan. They were new to physics when Dirac created his equation, and physicists were amazed that their familiar scalars, vectors and tensors, were not enough to model all aspects of reality — these new quantities incorporating spin properties were also required. Roger Penrose prefers a 2-spinor formulation to the more common 4-spinors used by physicists, but the two formalisms are mathematically equivalent. In his book *The Road to Reality*, Penrose offers his own perspective on the electron's inner behaviour. He splits the electron conceptually into two massless, light-speed, 2-spinor sub-particles called a 'zig' and a 'zag', which take it in turns to appear, the forward motion of the zig being continually converted into the backwards motion of the zag, and vice versa. The zig and zag are each the source of the other, and while the light speed velocity of the electron keeps reversing, its spin direction remains constant in the electron's rest-frame.

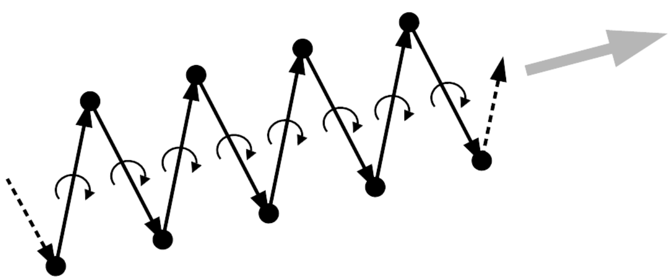


Fig. 13.4 Penrose's zig-zag electron.

Fig. 13.4 reproduces a diagram that Penrose has used to illustrate the electron's internal zig-zagging, along with its outer world motion through space in the direction of the grey arrow (deliberately exaggerated). Notice that the spin direction remains constant between the zig and zag legs of the motion. Penrose also regards each flip between zig and zag as a coupling to the Higgs field, which is the source of mass according to the standard model of physics. A particle's connection to the Higgs field only provides a very small part of its rest-mass, far more is due to the energy of its internal motions. (The Higgs 'particle' was inferred to exist from measurements in 2012 at the LHC collider, it has a lifetime of 10^{-22} second, shorter even than one zig or zag of the electron).

Throughout the history of physics it was taken for granted that the laws of nature were the same under mirror reflection. This law of parity conservation was known to be true for classical gravitation, electro-magnetism and the strong interaction, and it was assumed to apply also to the weak interaction that is responsible for radioactive disintegration. But in the mid 1950s Chen-Ning Yang and Tsung-Dao Lee suggested that the weak interaction might break this law, and experiments soon showed that they were correct. Another peculiarity of the electron's zig-zag is that only the zig particles take part in the weak interactions, zags, not at all.

> So are these zigs and zags real? For my own part, I would say so; they are as real as the 'Dirac electron' is itself real — as a highly appropriate idealized mathematical description of one of the most fundamental ingredients of the universe.
> Roger Penrose, *The Road to Reality*

Penrose's picture of the electron as a pair of zig and zag sub-particles endlessly morphing into one another is part of his own perspective on *zitterbewegung*.

Zitterbewegung

When Schrödinger played with Dirac's equation, he found an oscillation with a frequency of $2mc^2/\hbar$, or 1.55×10^{21} Hz, which he described as an interference between the electron's positive and negative energy states, the $2mc^2$ being the gap between the $+mc^2$ positive energy and the $-mc^2$ negative energy. This illustrates the tight coupling that exists between these two compartments of energy. Schrödinger also found an associated fluctuation of $\hbar/2mc$, or 1.9×10^{-13} metre, in the electron's position, and this is half its Compton radius. He called this fluctuation *zitterbewegung*, which translates as trembling motion.

In the ring model of an electron of Fig. 13.3, the centre of the electron's charge is assumed to be much smaller than the ring, and its motion at light speed around the ring provides a perpetual electric current that causes a magnetic field to thread through the ring with a magnetic moment equal to the Bohr magneton. The $\frac{1}{2}\hbar$ angular momentum of the electron's inherent spin derives from the motion of its charge around the ring, and its 'rest-mass' from the energy of this motion, with the centre of mass at the centre of the ring. The circumference of the electron's orbit in Fig. 13.3 is the Compton wavelength of an electron, and this same scale provides the stage where an electron performs its mysterious *zitter* dance. While mainstream physicists just regard the zitter as an unobservable quirk of the mathematics of Dirac's equation, over the years many physicists have offered speculations about what a zittering electron is up to, and a list of papers that I have found most informative (and not paywalled!) is included in the references. David Hestenes has even gone so far as to put the zitter dance at the very heart of quantum mechanics in his 1990 paper The *Zitterbewegung Interpretation of Quantum Mechanics*, justified by the fact that the Dirac electron and its internal motions provide a source for all the basic quantities and relations of quantum mechanics.

A magnetic moment describes the strength of a magnet, and the Dirac equation predicts that the electron should have a magnetic moment, described by a *g-factor*, of exactly 2, which results from the electron's $\frac{1}{2}\hbar$ angular momentum and a relativistic effect called Thomas precession. An anomaly arises because the experimentally measured value for the g-factor is very slightly larger than 2, and the *anomalous magnetic moment* $a = 1 + (g - 2)/2$ describes how much too big it is, a being just 1 if the g-factor is exactly 2. In 1948, Julian Schwinger used the nascent theory of quantum-electrodynamics, QED, to make the first approximate calculation of the electron's anomalous magnetic moment, finding that the Dirac electron's g-factor of 2 needed to be multiplied by a factor of $a = 1 + \alpha/2\pi$, where α is the same fine structure constant we saw above, and $\alpha/2\pi = 0.0011614$. This agreed well with experiments at the time, and Schwinger went on to win a Nobel along with Feynman and Tomonaga for his work on QED, the $\alpha/2\pi$ result being engraved on his tombstone. As we saw above, α^2 gives the ratio in scales between nucleus and atom, and the electron's Compton, or zitter scale is bang in the middle, at the geometric mean.

With the aid of Feynman diagrams to keep track of more of the infinite possibilities of interaction that quantum field theory allows, and faster computers, QED was later able to produce more accurate calculations of the electron's anomalous magnetic moment, and the most recent calculated value of a is $1 + 0.0011596\ldots$, which agrees with the experimentally measured value to more than 10 significant figures, making this the most accurately verified prediction in the history of physics. Recently, several researchers have questioned this, finding irregularities in the more recent experimental results that may be due to what is called *confirmation bias* — you tend to find what you expect to find.

The Helical Electron Model

The next step to improve on the ring model for an electron was a helical model, which attributes the zitter to an oscillatory helical motion hidden in the Dirac equation. This model has been investigated by several researchers, and is illustrated in Fig. 13.5. The helical model is similar to the ring model, but the ring is stretched out into a helix, and as the infinitesimal point of the electron's charge centre circulates around the helix at light speed, there is also a translational motion of its mass centre in space in the direction of the magnetic flux lines in Fig. 13.5.

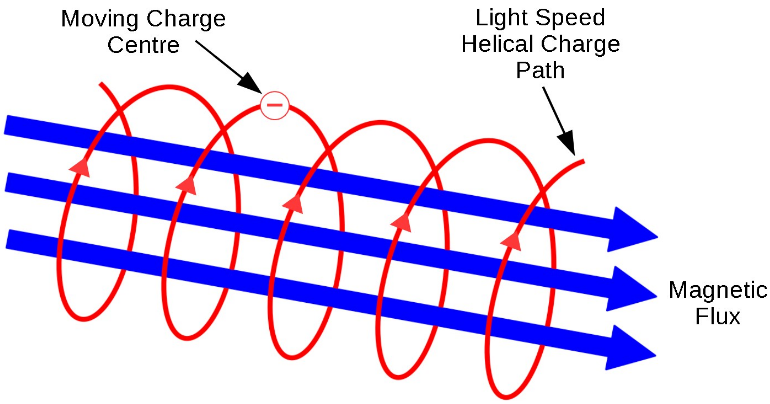


Fig. 13.5 The Helical Model of an electron.
The charge centre follows a helical path creating magnetic flux.
The electron moves in the direction of the magnetic flux.

There is nothing solid or stationary in this electron, its 'rest-mass' arises purely from the energy of the orbiting centre of charge, and the electric and magnetic fields that surround it, with its centre of mass following the middle of the helical path. In Chapter 7, we saw that whereas Mass, Length and Time are all affected by the speed of a particle according to relativity, charge remains unaffected, seemingly living in a privileged 'dimension' of its own, and free even to travel at light speed if it wants to.

If the electron is outwardly standing still in space, the helix is collapsed into a ring, and the centre of charge just goes around at light speed, but when the electron has translational motion in space, the helix is stretched out along the direction of travel. As the tangential motion of the charge centre is always at the speed of light c in this model, faster velocity through space requires slower rotational velocity. The geometry of this fixed tangential speed makes the rotational velocity v_r and translational velocity v obey a Pythagorean relationship:

$$v_r^2 + v^2 = c^2$$

so the rotational velocity is:

$$v_r = c\sqrt{1-(v/c)^2}$$

In the equations of special relativity, the Lorentz factor γ (which has no relation to the γ matrices that appeared in Dirac's equation above!) is written:

$$\gamma = 1/\sqrt{1-(v/c)^2}$$

Lorentz's γ can vary between 1 and ∞, and it is the multiplying factor that makes fast moving rulers shrink, clocks slow, and rest-mass m_0 increase to m, varying as $m = \gamma m_0$. We can use this gamma factor γ in the equation above for the rotational velocity of our electron's charge centre, and rewrite it as $v_r = c/\gamma$. This means that, as the electron's outer motion through space gets faster and γ increases, its charge centre's rotational velocity around the ring or helix must slow down.

It is quite remarkable that the baffling mass, length and time distortions of relativity can be reproduced in this model of the inner behaviour of an electron, its motion being shared between rotation and translation, and regulated by the constant speed of light. Could the inner zitter mechanism of electrons be the source of all the strange effects of relativity as well as those of quantum mechanics? Recall that it is only through electrons that we are able to observe anything at all — electrons rule the world.

The Toroidal Solenoid Electron Model

The next step in understanding the electron's inner structure was to wrap the helical model around into a donut shape and create the *toroidal solenoid electron model*. This was done in 1991 by a former student of Compton, Winston Bostick. Years earlier, in 1956, Bostick had discovered plasmoids, coherent and stable structures of plasma and magnetic fields that have since been used to explain such effects as ball lightning and magnetic bubbles in the ionosphere. Bostick's electron model is a kind of plasmoid.

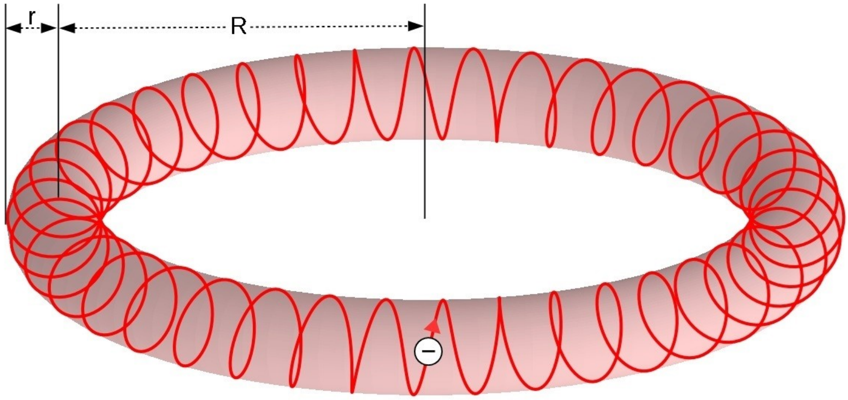


Fig. 13.6 Toroidal Solenoid model of electron.

In the toroidal solenoid model shown in Fig. 13.6, the electron's charge centre still moves at the speed of light, and the radius R of the torus is the same as the radius of the ring in the ring model. To complete an orbit, the charge centre now has to wrap N times around the small radius r of the helix, as well as going around the torus radius R. The longer distance travelled is equivalent to the radius of the ring model being increased by what Oliver Consa calls, in his 2018 paper *Helical Solenoid Model of the Electron*, a helical g-factor:

$$g=\sqrt{1+(rN/R)^2}$$

Because the angular momentum of the electron is fixed by being quantised, the rotational velocity around the ring must decrease by a factor of g to compensate for the factor of g increase of radius, and thus leave the angular momentum unchanged. But in calculating the magnetic moment, the equivalent radius increase appears as a factor g^2, and this results in a nett increase of the magnetic moment by a factor g. So we then have a simple physical cause for the electron's anomalous magnetic moment, and its value can be calculated from the toroidal solenoid model's geometry, with R being the zitter radius and r a free parameter. Consa finds a helical g-factor of $g=\sqrt{1+\alpha/\pi}$ = 1.0011607, closer to the experimental value than Schwinger's original result.

The Helical Solenoid Electron Model

Just as the earlier ring model gave way to the helical model when an electron moves through the space outside itself, our normal world, the toroidal solenoid model gives way to the helical solenoid model of Fig. 13.7 when the electron stops marking time on the spot, and sets out on a path through space in the form of a spiral within a spiral.

Consa was able to predict from this model that, contrary to the predictions of QED, the electron's g-factor has a very high frequency oscillatory component. In 2005, the wavelength corresponding to this frequency was detected experimentally in silicon crystals, and this has provided strong evidence that the family of zitter models described above are on the right track.

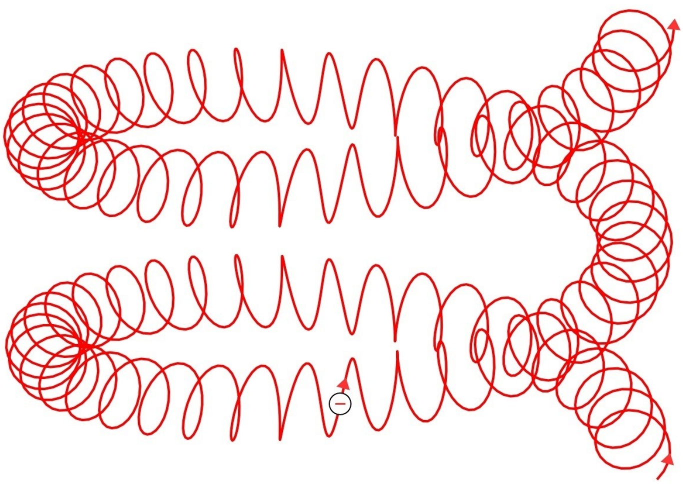


Fig. 13.7 The Helical Solenoid Model of an electron.

A Quantum Oscillator

The simplest form of oscillator in electronics is an LC circuit, where L is inductance and C is capacitance. In a perfect LC circuit without resistance, energy cycles back and forth between storage in the magnetic field of an inductance coil, and the electrical field that exists between the charged plates of a capacitor. There is a good animation of this here:

https://upload.wikimedia.org/wikipedia/commons/8/80/Tuned_circuit_animation_3_300ms.gif

Although nothing can be seen to move in the invisible flux of electric and magnetic fields, they do carry energy and momentum, just like a moving object. In one cycle of an LC circuit, each form of energy, electric and magnetic, has its maximum when the other is zero, following the same Pythagorean relationship of sine and cosine functions that we saw in Chapter 4. LC oscillators provide standard clock frequencies clocks for many applications, and often incorporate a precisely cut quartz crystal to stabilise the frequency.

In the ring model of Fig. 13.3, the ring behaves as a superconductor, and so allows the charge centre to circulate endlessly around the ring without resistance. It is known from macroscopic experiments that when charge flows around a superconducting ring, the magnetic flux due to the electric current is quantised, this flux quantum being yet another facet of the Planck. The charge centre of an electron carries a quantum of electric charge, and its motion provides a quantum of magnetic flux. Consa and others have shown that the rest-mass energy of an electron can be expressed as the sum of its inductive magnetic energy and capacitive electric energy, the two contributions oscillating at the zitter frequency. But which comes first, which is the source, the electric charge quantum or the magnetic flux quantum, or neither?

> … the electron is formed by two indivisible elements; a quantum of electric charge and a quantum of magnetic flux, the product of which is equal to Planck's constant. The electron's magnetic flux is simultaneously the cause and the consequence of the circular motion of the electric charge.
> Oliver Consa, *Helical Solenoid Model of the Electron*

In his 2020 paper *Zitterbewegung structure in electrons and photons*, David Hestenes sees the electron as a 'quantum oscillator that serves as a digital clock in physical processes', and he further identifies the electron clock as 'the fundamental mechanism grounding the units of action and charge'. We know that almost all computers require a precise master clock to synchronise the operations of all their logic gates, and the Cosmic Computer also needs precise clocking, but it uses a distribution of around 10^{80} standard electron clocks, each ticking at precisely the same rate in its own reference frame.

> The rotation time period of the electron acts as the electron's internal clock. As a result, although there is no absolute time in the universe, each electron is always set to its proper time. This proper time is relative to the electron's reference frame and its velocity with respect to other inertial reference frames.
> Oliver Consa, *Helical Solenoid Model of the Electron*

Other Ideas

In his 2012 paper *Two-vortex structure of electron, nonlocality and Dirac equation*, S. C. Tiwari finds that the electron's internal structure consists of an almost point-like vortex rotating around a core vortex. He also cites other work which emphasises the implicit time non-locality in the Dirac equation, and interprets the ±c light speed motions of zitter as the oscillations between the past and future of a spatially extended particle that is forced to appear as a point particle in the Dirac theory. It seems that time flows forwards and backwards within the unit bubble of space and time that hosts the zitter cycle — tiny pieces of past and future coexist in this zitter bubble.

Reversing the direction of time measured by something rotating, like the hands of a clock, is easy — you just reverse the direction of rotation. At the macroscopic scale at which we live, we only see the one-way arrow of linear time pointing outwards from its supposed source at the big bang Planck scale singularity, towards the Hubble scale boundary of the Universe. This one-way nature of time has long been a mystery, but the most common explanation invokes the insistence of the second law of thermodynamics on entropy increase, energy always trying to spread itself out, reduce its temperature, dilute itself into more and more bits of information. But while the outer world has space for entropy increase, there does not seem to be room for it within atoms. Penrose does not describe his zig and zag particles as going backwards and forwards in time, but this seems to be a valid viewpoint, and it also seems deeply connected with the mystery of why the laws of physics are symmetric under time reversal at small scales.

In his 2020 paper, Hestenes builds on the work of Consa and others, describing the electron as a point singularity on a lightlike toroidal vortex. He explores many other ideas including the possibility that a quantum of charge is a topological distortion of space-time, and that an electron is a fundamental singularity of the vacuum. He also gives a possible answer to the obvious question about the various ring models:

> "What holds the ring together?" Remarkably the obvious answer "Gravity!" has strong independent support from General Relativity. The main fact is that the celebrated *Kerr-Newman solution* of Einstein's equation involves a charged ring singularity with spin and g-factor just like our zitter model of the electron.
> David Hestenes 2020, *Zitterbewegung structure in electrons and photons*

On the subject of Schwinger's QED calculation of the electron's anomalous magnetic moment he says:

> Conventional QED provides no physical explanation whatsoever for the Schwinger result, despite its mathematical sophistication and claims for unprecedented accuracy. Indeed I heard Feynman himself declare that such lack of physical insight would not be tolerated in any other branch of physics.
> David Hestenes 2020, *Zitterbewegung structure in electrons and photons*

The papers of Jean Maruani, listed in the bibliography, are full of insights about the inner world of the electron, with many ideas going back to de Broglie's book *l'electron magnetique*, an English .pdf version of which is also listed in the bibliography. Most of these ideas have been ignored by mainstream physicists, but are now being reexamined as a result of the lack of progress in fundamental physics since the 1980s. Maruani not only sees the zitter scale as sitting at the geometric average of an electron's classical electrical radius and the Bohr radius of an atom, with the fine structure constant α providing the invariant scaling factor up or down, but he also finds that the zitter scale is the geometric average of gravitational curvature inside and outside the electron, with these obeying another invariant scaling factor. Several physicists have suggested that there is a strong connection between gravity and spin, both having non-local properties, and this might provide the key to understanding the non-local sharing of information in quantum entanglement.

There have been other suggestions as to what keeps the electron's charge centre in orbit at the Compton zitter scale and prevents it from just flying off. In his 2013 paper, Maruani gives an intriguing description of the electron dancing the zitter around its virtual anti-particle image viewed in a mirror:

> Zitterbewegung is what relates a *real* electron and a *virtual* positron, their mass and spin being linked by a wave beat between the two states…
>
> What maintains the *massless charge* -e in a spinning orbit and provides it with the energy accounting for the electron rest mass would then be the *electric potential V* exerted by an opposite charge at a distance equal to twice the Compton radius as if the spheres of the electron and its mirror image were contiguous.
> Jean Maruani 2013, *The Dirac Electron as a Massless Charge Spinning at Light Speed: Implications on Some Basic Physical Concepts*

De Broglie also conjectured that a photon results from the ‘fusion’ of an electron and its positron anti-particle, the ½$\hbar$ spins of the electron and positron adding together to provide the $\hbar$ spin of the photon. This proposal makes sense if you remember that when an electron and positron meet, they annihilate one another, their opposite charges disappear and the result is just — photons. Hestenes (2020) models the photon as an electron-positron pair trapped in a vortex with energy proportional to the photon frequency.

Summary

So the zitter dance appears as a mathematical curiosity in mainstream physics, the zigs and zags of a morphing particle according to Roger Penrose, a trembling motion at the speed of light for Erwin Schrödinger, a quantum electromagnetic LC oscillator from an engineer’s perspective, and the key to quantum mechanics in the *zitterbewegung interpretation* of quantum mechanics proposed by David Hestenes. The cyclic exchanges of energy that take place in the zitter can be seen as a wave-beat between positive and negative energies or masses, LC oscillations of electric and magnetic energy, or exchanges between kinetic and potential energy. Recall that the least action principle requires the balance of the latter two forms of energy, and the Universe itself is a balance of positive and negative energies that add up to nothing. The energy of the light speed motions within an electron only shows up in the outer world as its ‘rest’ mass.

Mainstream physics has dismissed the internal makeup of an electron as unmeasurable and therefore to be ignored, but has also never provided an understandable explanation of why energy stays bottled up in electrons, protons, and atoms, but leaks out of everything else. We are told that this is ‘explained’ by quantum mechanics, but quantum mechanics does not provide explanations, it is only a set of axioms and rules of calculation that pass all experimental tests, yet lack any further justification.

14
Gravity

Newtonian Gravity

Isaac Newton's 1680 law of universal gravitation was one of the greatest advances in the history of science. It says that every particle in the Universe attracts every other particle, with a force that is directly proportional to the product of their masses, and inversely proportional to the square of the distance between their centres. It enabled accurate predictions to be made for the orbits of planets in the solar system, as well as the behaviour of falling objects on Earth. When Edmond Halley applied his friend Newton's theory to comets, he was able to correctly predict the return date of the comet that now bears his name. When slight deviations from Newtonian predictions were found for the orbit of Uranus, Urbain Le Verrier was able to deduce the existence and location of an unobserved planet. The new planet, Neptune, was found by the Berlin observatory within 24 hours of receiving Le Verrier's data. Newton's theory was spectacularly successful for hundreds of years, and continues to work today in most situations — nothing better was needed to land humans on the moon in 1969.

General Relativity

In 1905, when Newton's law of gravity was found to be incompatible with his new theory of special relativity, Einstein started looking for a more fundamental law. He found his equations of *general relativity* in 1915, and they predicted two very small deviations from Newton's theory — the precession of the orbit of Mercury, and the deflection of light rays passing close to the Sun. When Einstein used his new equations to calculate the precession of Mercury, and found the correct observed value which is half that of Newton, he experienced a great high that lasted for several days. In 1919, when Eddington's expedition took advantage of a solar eclipse to observe the deviation of starlight passing close to the sun, they found Einstein's theory correct, and Einstein became a star of science.

In 2015, the centenary year of their prediction by Einstein's general theory of relativity, gravitational waves were detected by the LIGO experiment. The black holes, whose violent merger was the source of the gravitational waves, were also a prediction of general relativity. Nevertheless, gravity remains poorly understood.

In comparison with the electrical force between a pair of charged particles, the gravitational force is 10^{40} times weaker, so there is no way to measure the gravitational attraction between two particles, and the smallest range over which gravity has been measured is about 1 millimetre, so there is no certainty that it even operates below this scale. While general relativity has passed all experimental tests, it still presents problems, the greatest one being that it is incompatible with quantum mechanics, the 20th century's other great theory of physics, and for nearly a century, all efforts to marry the two theories together have failed. Freeman Dyson showed in his paper *Is a Graviton Detectable?* (2012), that if there is a quantum unit of gravity, a *graviton*, it is almost certainly undetectable. Subtle indeed is the force that shapes our Universe.

In contradiction to special relativity, quantum mechanics relies, like Newton, on a universal time that is shared by all particles — how else could particles maintain the quantum mechanical phase relationships between their inner clocks when they move relative to each other? The problem is avoided if we switch from special relativity to Lorentzian relativity, which allows a Now, as we saw in Chapter 8. The standard way that physicists dodge this problem is by upgrading from quantum mechanics to quantum field theory, but this has its own contradictions, like the little one of predicting that empty space has an energy density 10^{120} times greater than the observed value.

General relativity also predicts the existence of a singularity at the big bang origin of the Universe, and also at the centre of every black hole, an infinitesimally small point to which all the mass must inevitably descend. Most physicists, including Einstein, found this obnoxious, and the existence of black holes was not generally accepted until the 1960s. More recently, LIGO's recordings of black hole mergers, and images of the giant black hole at the centre of the Milky Way, have strengthened the evidence for the existence of black holes that are heavier than our Sun, but no evidence has been found yet for smaller black holes like the ones predicted by big bang theory.

Many other theories of gravity have been proposed in the last 100 years, often as extensions to general relativity — a few promising ones are described next.

Einstein-Cartan Gravity

Einstein's essential insight for general relativity is very simple — the presence of matter or energy has a volume reducing effect on the region of space it occupies, this is what we call gravity — gravity sucks. While the tensor mathematics of general relativity is forbidding enough to non-mathematicians, it is still insufficient to fully describe space-time. Besides being subject to 'curvature' from the presence of energy-matter (imagine the axes bending), it turns out that space-time can also suffer from torsion (imagine the axes twisting) which the tensor formalism does not allow for. General relativity assumes space-time to be torsion free, and this is a good assumption, as torsion only becomes important at very small scales and high energies which are experimentally inaccessible.

The intrinsic spin of particles was unknown in 1915 when Einstein published his equations of general relativity, and was only discovered in the mid 1920s. Elie Cartan was inspired by Einstein's work, and in 1922 he extended the Riemannian geometry of general relativity to include the torsional properties of space-time via new, tensor-like mathematical entities that he called *spinors.* Wolfgang Pauli used spinors to describe the 2 state, up-or-down spin behaviour of particles in 1927, and the Pauli spin matrices (see Chapter 4) are the simplest examples of spinors, describing the rotations of a 2 state system in 4D space-time. As we saw in Chapter 13, Paul Dirac found it necessary in 1928 to double the number of states from 2 to 4 in order to construct his great equation that correctly describes the quantum behaviour of an electron even at high relativistic speeds, unlike the Schrödinger equation. Dirac's spinors are 4 component entities, and they remain mysterious…

> "No one fully understands spinors. Their algebra is formally understood but their general significance is mysterious. In some sense they describe the "square root" of geometry and, just as understanding the square root of -1 took centuries, the same might be true of spinors"
> Sir Michael Atiyah, Fields medallist

Recall that i, the square root of -1 manifested itself in the √NOT quantum logic gate in Chapter 9.

In his 1918 book, *Space Time Matter*, Hermann Weyl laid out the mathematical structure of space-time, based on the ancient theorem of Pythagoras, and Riemann's multi-dimensional line element that we saw in Chapter 4. Weyl was laying the foundations for an attempt to merge the theories of gravity and electromagnetism. This attempt ultimately failed, as Weyl acknowledged in later editions of *Space Time Matter*, partly because, as Einstein had pointed out, Weyl's line element was no longer an invariant quantity, and so the lengths of rods and the readings of clocks would come to depend on their history. A few years later, Weyl's ideas were found to be useful, provided that his real numbers were replaced by complex numbers, and this gave rise to the 'gauge' theories that now play a central role in physics, and offer possible routes to a theory of quantum gravity. Gauge theories take advantage of symmetries that allow one thing to change while everything else remains the same.

space-time is both a *metric* space and an *affine* space. Metric space, as its name suggests, has a measure of *distance* between any two points, whereas an affine space has no unit of measure, and only keeps track of the *directions* that vectors point in. It is part of the magic of the Pythagorean relationship that it can provide both a unit of measure in its invariant 'line element', and also keep track of relative angles to map directions. The line element is allowed to be multi-dimensional, and in the case of rotations, it represents an infinitesimally small, invariant unit of volume.

The underlying general feature that determines the metrical structure at a point P is the group of rotations.

> … the group of rotations consists of all linear transformations that convert the quadratic groundform into itself. But the group of rotations need not have an invariant at all in itself (that is, a function which is dependent on a single arbitrary vector and which remains unaltered after any rotations).
>
> … Since a rotation is "not to alter" the vector body, it must obviously be a transformation that leaves infinitesimal elements of volume unaffected.
>
> … The group of rotations that does not vary with position exhibits a property that belongs to space as a form of phenomena; it characterises the metrical nature of space. The metrical relationship, from point to point, however, is *not* determined by the nature of space, nor by the mutual orientation of the groups of rotation at the various points of the manifold. The metrical relationship is dependent rather on the disposition of the material content, and is thus free and capable of any "virtual" changes.
> Hermann Weyl, *Space Time Matter*

In the above, Weyl's 'quadratic groundform' is the Pythagorean relationship that provides an invariant line element, and he points out that rotations are free from the 'curvature' of space-time that is caused by the presence of matter.

Riemannian geometry was described in Chapter 4, and uses a 4-dimensional, Pythagorean 'line element' as its unit of measure for tracking the distortions of the space-time metric which account for gravity in general relativity. A metric relies on a standard unit of measure that is shared between different dimensions, but there is another type of relationship between dimensions that has no sense of measure but only cares about the *directions* in which vectors point; this is called an *affine connection*, and it comes in two versions: symmetric and anti-symmetric. General relativity uses only a symmetric affine connection, the anti-symmetric part that allows for torsion is ignored, and this makes no difference for calculations, except when studying something like the Universe just after the big bang, when the energy density was enormous.

Following on from attempts by Weyl and Eddington to unify gravity with electromagnetism, Einstein worked on his own extended, *teleparallel* theory of gravity in 1928, as just one of the many attempts he made during the rest of his life to find a unified theory. Einstein was enthusiastic about *teleparallelism* (also called distant parallelism or absolute parallelism), and someone must have told the press.

> On November 4, 1928, The New York Times carried a story under the heading 'Einstein on verge of great discovery; resents intrusion,' followed on November 14 by an item 'Einstein reticent on new work; will not "count unlaid eggs."'
> Abraham Pais, *Subtle is the Lord*

A few months later Einstein had to go into hiding, so great was the world's excitement about his new theory, and it had now run into problems... Teleparallelism requires a special kind of nonsymmetric connection to bring electro-magnetism into the equations of general relativity, and this made the curvature tensor vanish — Einstein's own source for gravity was gone! Eddington, Weyl and Pauli all criticised Einstein's new theory, and he had no good answers. After a few more efforts, Einstein gave up, and later described teleparallelism as a wrong path, although he continued to play with theories that used a nonsymmetric connection, and so able to include torsion. Several other physicists have continued developing general relativity to include space-time torsion, and the result is now called the Einstein-Cartan-Sciama-Kibble, E-C-S-K, or *metric-affine* theory of gravity.

In his paper *Intrinsic spin requires gravity with torsion and curvature* (2013), Nikodem ł shows that teleparallel gravity is inconsistent with the conservation of total angular momentum. He also shows that in curved space-time, the existence of particles with intrinsic spin *requires* that the anti-symmetric part of the affine connection is not zero, and so torsion must be present. As all the fundamental particles of matter have spin built in, space-time must also therefore possess torsion. In other papers he investigates further how spin and angular momentum interact with space-time.

> Spin interacts with space-time and endows it with a geometric property called torsion. The effects of torsion are comparable to curvature only at extremely high densities of matter. To visualize torsion, space-time can be imagined as an elastic, thin cylindrical rod. Bending the rod corresponds to curving space-time, and twisting the rod corresponds to space-time torsion. If a rod is thin, it can be bent, but it is hard to see if it is twisted or not.
> Nikodem Poplawski, *Cosmological consequences of gravity with spin and torsion*

Nonlocal Gravity

In general relativity, the warping of space-time depends on the amount of energy and matter present, and this is represented by an energy-momentum tensor. But this tensor does not include the energy of the gravitational field itself, which also contributes to gravity, and this greatly complicates the mathematics because the equations become *non-linear*. So how does general relativity cope with this? According to Roger Penrose in *The Road to Reality*, the gravitational contributions of energy-momentum "…in a sense, slip in through the cracks…" that separate the *local* equation from an integral conservation law of *total* energy-momentum. Gravity is an inherently non-local force, and Penrose also sees the non-local nature of gravity as one of the main problems when looking for a quantum theory of gravity.

The energy of a gravitational field is not only non-local, but also *negative*, unlike the *positive* energy that is contained in the rest-mass of particles, and the kinetic energy of their motions. This is how the Universe can arise from nothing, the two opposite energies can both grow provided that they always cancel each other out. Alan Guth gives a simple explanation of why gravitational field energy is negative in Appendix A of his book *The Inflationary Universe*. Several other physicists have also described a zero energy Universe, e.g.: Stephen Hawking in *A Brief History of Time*, and Lawrence Kraus in *A Universe from Nothing*. As we saw in Chapter 6, the least action principle governing all physical interactions is enforced by balancing kinetic and potential energy — there appears to be a deep significance in the counter-balancing of two forms of energy.

In his book *Nonlocal Gravity*, Bahram Mashhoon describes a particular nonlocal generalisation of general relativity that he and many co-workers have developed, and briefly mentions several other formulations by other physicists. Relativity theory, whether special (without gravity), or general (with gravity), is based on the postulate of *locality*, meaning that the measured value of a physical quantity at any event in space-time is directly influenced only by quantities in its immediate neighbourhood. We saw in Chapter 4 how a Pythagorean 'line element' provides the mathematical unit of measure for such local interactions. But while local point-like interactions may be appropriate for particles, they are not appropriate for electromagnetic waves which are inherently non-local.

> …Bohr and Rosenfeld pointed out in 1933 that, as a matter of principle, classical electric and magnetic fields cannot be measured instantaneously, and only their space-time averages have immediate physical significance. Thus in the process of field measurement, the past history of the observer must be taken into account. This issue acquires added urgency when one recognizes that accelerated observers are endowed with intrinsic invariant length and time scales associated with their motions. The acceleration scales can be constructed from the speed of light and the appropriate magnitudes of the observer's translational and rotational accelerations.
> Bahram Mashhoon, *Nonlocal Gravity*

Recall that we saw in Chapter 7 how Maxwell's equations of electro-magnetism describe the interdependence of the rotational magnetic curl, with scalar electric divergence. The *translational* and *rotational* components of acceleration are also separate, but related entities. The coupling of the inherent spin and orbital spin of an electron in an atom was mentioned in Chapter 13, and Mashhoon and his co-workers used the same phenomenon of spin-rotation coupling to develop an acceleration function that takes account of the past history of an observer. This allowed them to develop a nonlocal extension of special relativity which they published in 2008. Mashhoon has more recently extended this work to general relativity.

The *equivalence principle* says that the *gravitational mass*, which describes the gravitational force that a mass feels, or exerts on other masses, is equal to its *inertial mass*, its resistance to the acceleration caused by an applied force. This was the key insight that came to Einstein in a lightbulb moment while he was working in the patent office in Bern, and helped guide him in the development of general relativity.

> It is important to observe that Einstein's principle of equivalence loses its operational significance if one does not know a priori what accelerated observers in Minkowski space-time actually measure; that is, the locality postulate

> plays a crucial role in Einstein's extension of relativity theory to the gravitational domain. In nonlocal special relativity theory in Minkowski space-time, an accelerated observer in general carries the memory of its past acceleration. Invoking Einstein's original insight in this more general circumstance, one expects that gravity should be nonlocal as well, and in nonlocal gravity, the gravitational memory of past events must be taken into account.
> Bahram Mashhoon, *Nonlocal Gravity*

Special relativity uses ideal inertial observers in Minkowski space-time which is 'flat' with no curvature, gravity is turned off. Anything that has mass qualifies as an observer, including just a particle of matter. But while the laws of non-gravitational physics have all been formulated with respect to such ideal *inertial* observers, the experimental evidence all comes from *accelerated* observers — gravity is just acceleration, and anything in orbital motion experiences acceleration.

> Simply stated, the fundamental microphysical laws, such as the principles of quantum mechanics, have been formulated for nonexistent ideal inertial observers, while all actual observers are accelerated. The resolution of this dichotomy requires an a priori axiom that relates inertial and accelerated observers. The observational consequences of such an axiom should then be compared with experimental results.
> Bahram Mashhoon, *Nonlocal Gravity*

The metric tensor of general relativity has ten components that describe gravity as the 'curvature' of space-time. To create a nonlocal extension to general relativity, Mashhoon introduces *fundamental observers* which carry sixteen components to fully describe the gravitational field. This requires a new Weitzenböck 'connection', besides the standard Levi-Civita connection of general relativity. The Levi-Civita connection is torsion-free but gives rise to the curvature of space-time, whereas the Weitzenböck connection is curvature-free but has torsion.

> At each event in space-time, the curvature and torsion tensors both characterize the gravitational field. In fact, the curvature of the Levi-Civita connection and the torsion of the Weitzenböck connection are complementary representations of the gravitational field in extended GR.
> Bahram Mashhoon, *Nonlocal Gravity*

Entropic Gravity

In his paper, *On the Origin of Gravity and the Laws of Newton* (2010), Erik Verlinde describes a way of understanding gravity based on information. Verlinde sees gravity and space-time geometry as *emergent* phenomena, arising from the properties of information. The Holographic Principle was described in Chapter 12, and asserts that all the information describing the contents of any 3D volume is 'written' on its 2D boundary. Entropy is another name for the information carrying capacity of a system. In Verlinde's theory, there is an *entropic force* that originates in a system with many degrees of freedom, driven by the statistical tendency of its entropy to increase in obedience to the second law of thermodynamics.

The laws of black holes are analogous to the laws of thermodynamics, so according to the holographic principle, a black hole, or any other region of space, not only has entropy, but also a surface temperature that decreases as the surface area increases. Some of these results were illustrated in Chapter 12. Bill Unruh and others showed that an accelerating observer experiences a rise in the temperature of its surroundings, and found an equation that relates acceleration and temperature. With these ingredients, and Newton's second law, which says that force equals mass times acceleration, Verlinde is able to derive Newton's law of gravity in a surprisingly simple fashion, and he goes on to derive Einstein's equation of gravity.

Dark Matter

The outer stars of a galaxy orbit much faster than they should if they only experience the gravitational attraction of the visible stars and matter in the galaxy. This 'missing mass' problem was first observed by Fritz Zwicky in 1933 for clusters of galaxies, and was later found by Vera Rubin in spiral galaxies. The favourite explanation is the presence of an invisible dark matter 'corset' that surrounds a galaxy and provides the extra gravity. Over the past sixty years, numerous candidate particles for dark matter have been suggested and rejected, with many ideas depending on *supersymmetry*, which postulates a new partner particle for every particle currently known to physics. But the LHC, our most powerful collider, has found no sign of any of these supersymmetric particles, and now, many physicists are looking at other ideas.

In 1983, Mordehai Milgrom suggested that the Newton's laws of gravity should be modified, rather than invoking dark matter — his theory is called Modified Newtonian Dynamics, MOND. According to Newton, the speed at which stars travel in their galactic orbits should decrease with distance from the galactic centre. But observations show that the speeds of outer stars stay about the same as those further in. MOND postulates that Newton's law is modified when the acceleration due to gravity drops below a critical value a_0. The value he found for a_0 to best fit the data is around 1.2×10^{-10} metres/sec^2. This is very small, but almost enough to accelerate something from rest to the speed of light if it was maintained for the 10^{17} seconds since the big bang. While both MOND and the dark matter theory agree well with the data in most cases, they each also fail in some cases, so the jury is still out.

There are many other approaches to explaining dark matter without needing particles. In his papers *Cosmological consequences of gravity with spin and torsion* (2013), and *Non-particle dark matter from Hubble parameter* (2019), Poplawski shows how space-time torsion may help solve the mystery of dark matter. In his book *Nonlocal Gravity*, Mashhoon sees the dark "source" of gravity as simply a manifestation of the nonlocal aspect of the gravitational interaction.

Dark Energy

Normal matter accounts for less than 5% of the mass in the Universe, and dark matter provides a further 27%, and both types generate normal gravity. The remaining 68% is called *dark energy*, and it generates anti-gravity, and is used to explain the accelerated expansion of the Universe at large distances. Dark energy is also equivalent to the *cosmological constant* that Einstein added to general relativity to prop the Universe open and prevent it from collapsing. He added it after it was pointed out to him that his original equation left the Universe unstable.

In his papers *Affine theory of gravitation* (2018), and *Non-particle dark matter from Hubble parameter* (2019), Poplawski shows that the cosmological constant as dark energy can arise naturally in the simplest description of the gravitational field when there is no matter present, but only torsion. In his view, the observed accelerating expansion of the Universe may be the strongest evidence for space-time torsion.

There are also numerous other theories being developed to account for dark energy.

Singularities

At the smallest scale, the Planck scale of 10^{-35} metre, physics breaks, and physicists do not know how to fix it. This problem also limits attempts to wind time back to the big bang, when the Universe is supposed to have originated at the Planck scale. It is equivalent to trying to divide by zero, a computer either refuses to do it, or gets stuck in an eternal loop and crashes — it cannot be done. There are other types of singularity. A single pixel in a digital picture is a sort of singularity, because it is the smallest possible component of the picture, and it cannot be divided. The Compton scale of a fundamental particle like the electron is also a singular limit, but in that case, as we saw in Chapter 13, the Dirac equation does at least allow us to take a mathematical peek inside.

The E-C-S-K extension of general relativity to include torsion that we saw above provides another way to avoid a big bang singularity, and this is being explored by several researchers.

> … torsion manifests itself as a force that counters gravitational attraction. … in the ultrarelativistic early Universe… torsion seems to prevent the collapsing spin-fluid matter from reaching a singularity.
> Nikodem Poplawski, *Cosmological consequences of gravity with spin and torsion*

String theory also dodges the singularity problem that comes from using infinitely small point particles, by replacing them with extended bits of string, and in his book *The Elegant Universe*, Brian Greene describes how string theory can transcend the Planck length. There are two basic modes that a string can take, wound (heavy), and unwound (light). If you imagine a garden hosepipe of radius R, an unwound string measures along the hosepipe in units of R, and gets heavier in proportion to R, whereas a wound string measures around the hosepipe, and gets heavier in proportion to 1/R, counting the number of times it wraps around the pipe.

> … as R—the quantity measured by unwound strings—shrinks to 1 and continues to get smaller, 1/R—the quantity measured by wound strings—grows to 1 and gets larger. Therefore, if one takes care to always use the light string modes—the “easy” approach to measuring distance—the minimal value encountered is the Planck length.
>
> In particular, a big crunch to zero size is avoided, as the radius of the Universe as measured using light string-mode probes is always larger than the Planck length. Rather than heading through the Planck length on to even smaller size, the radius, as measured by the lightest string modes, decreases to the Planck length and then immediately starts to increase. The crunch is replaced by a bounce.
> Brian Greene, *The Elegant Universe*

Another way of avoiding the big bang singularity is Penrose's *Conformal Cyclic Cosmology* that he describes in the book *On Space and Time*.

Is Gravity the Great Decision Maker?

As we saw in Chapter 11, quantum mechanics provides the mathematical machinery to calculate probabilities for the outcomes of measurements on quantum systems, but no more, and an essentially random element remains — the dice throwing that Einstein despised, and led him to maintain until his death that quantum mechanics is incomplete. Many attempts have been made to find a physical mechanism that can select quantum measurement outcomes deterministically, and return physics to the righteous path of cause and effect, and these are generally known as hidden variable theories. Roger Penrose, along with many other physicists, singles out gravity as the decisive force that is responsible for selecting the outcomes of quantum measurements.

> It is my own standpoint, with regards to quantum state reduction, that it is indeed an objective process, and that it is always a gravitational phenomenon.
> Roger Penrose, *The Road to Reality*

We looked at David Bohm's version of quantum mechanics in Chapter 11, and noted his view that there is a periodic inner process in the Universe that provides an inner clock. The zitterbewegung, described in Chapter 13, seems to be the prime example of this.

> We are appealing to the notion that a particle has a rich and complex inner structure which can respond to information and direct its self-motion accordingly.
> David Bohm & Basil Hiley, *The Undivided Universe*

Bohm also introduced his concept of *active information*.

> We have … introduced a concept that is new in the context of physics—a concept that we call *active information*. The basic idea of active information is that a form having very little energy enters into and directs a much greater energy. The activity of the latter is in this way given a form similar to that of the smaller energy.
> David Bohm & Basil Hiley, *The Undivided Universe*

In the 21st century, with so many large machines and processes controlled by the activity of just a few tiny electrons in a silicon chip, this now seems rather obvious

— Bohm's active information is at work everywhere in the modern world. So why would the Universe be any different, especially after considering the vast concealed computing power that we now know to be contained within all matter?

Einstein's objection to the dice throwing randomness of quantum mechanics is well known, but it is less well known that he was responsible for introducing an essential randomness into physics himself in 1916, as part of his theory describing the emission of photons from atoms, work that later led to the development of lasers. A photon is emitted when an electron drops from an excited state in an atom, to a state of lower energy, but the direction the photon chooses to take appears completely random. It seems surprising that it is possible to concentrate photons emitted in such a way into a tightly directed laser beam, but this is done by constraining the emitted photons to only exit the laser through a narrow aperture. The random choice of photon emission direction is another example of a causeless effect in physics, but it seems only logical that the complicated phase relationships within an atom must play a part in choosing the launch direction of a photon.

One of the problems with finding a deterministic mechanism for quantum outcomes is balancing the energy books — energy is conserved, it cannot come from nowhere or disappear. As we noted in Chapter 9, computing using particle spins takes very little energy, so one of the attractive features of gravity with space-time torsion is that it might leave a door open for gravity to control quantum outcomes. In *The Cosmological consequences of gravity with spin and torsion*, Poplawski calculates that the effects of torsion only become significant for a neutron when the mass-energy density of space reaches 10^{45} kg/m^3. The mass-energy density of a neutron or proton in an atom is about 10^{17} kg/m^3, which is 10^{28} times less, so even though torsion must be present, it is vanishingly weak in normal matter. But gravity is all about many particles acting in unison to reduce the volume of the space they inhabit, sucking in clock cycles of time, and shortening rulers, and it takes as many as 10^{20} protons to make up a single Planck unit of mass. Could the combined effect of space-time torsion in many particles be enough to steer quantum decision making? Unfortunately, I have not been able to find any discussion of the possibility that space-time torsion plays a part in deciding quantum outcomes, so who knows, but perhaps each actual event is just a simple twist of fate.

Does Local Contain the Non-Local?

David Bohm described the primary process of the Universe as an oscillation, a breathing in and out, or enfolding and unfolding, between his dual, implicate and explicate orders. The same idea has been presented in this book as the dual representation of the Universe's information content across inner and outer domains of scale, the two being kept in synch by the continual NOT operations which the Dirac equation describes, and the Margolus-Levitin theorem counts. This duality of information means that the world outside a particle has a dual representation within it, so the *nonlocality* of gravity may be seen as just a symptom of the *localisation* of gravitational information processing *within* each particle, sharing the world of spin and rotation with all other particles.

Several other mysteries of physics are also understandable from this dual perspective. The many-worlds interpretation of quantum mechanics requires countless alternative universes to exist, it has been described as "cheap on assumptions, but expensive on universes", so where are all these universes? Perhaps their information is represented *inside every particle*, in a superposition of all the possibilities that have not manifested out into true reality. Reality is truth.

Seth Lloyd has calculated from thermodynamic laws that the Universe contains 10^{92} bits, most of this being carried by photons, which greatly outnumber the 10^{80} or so matter particles. In *Programming the Universe*, Lloyd also suggests that the remainder of the 10^{122} bits that are represented holographically on the boundary of the Universe, belong to gravitational degrees of freedom that we do not yet understand. This would mean that there are 10^{30} gravitational bits for every single bit that we are able to recognise and count, and the pool of gravitational information would dwarf our real world information. These gravitational bits also seem to transcend the Scale dimension in a non-local manner, threading from the 10^{122} bit boundary, down through particles to a common holographic projection centre of the Universe at the Planck scale.

Isaac Newton was never happy with the instantaneous action at a distance that his theory of gravitation relied on, it had the smell of witchcraft, and while he was fortunate not to be under the direct control of the Roman Catholic church like Galileo, it was still a dangerous idea in the England of Newton's lifetime. Faraday invented the concept of an invisible 'field' in all the space that surrounds matter to describe electro-magnetism, and Maxwell chose to develop Faraday's idea partly because an electromagnetic field can provide the means of communication for electromagnetic effects through space, while theories due to Weber and others had to rely on action at a distance.

> It seems at first sight natural to explain the facts by assuming the existence of something either at rest or in motion in each body, constituting its electric or

> magnetic state, and capable of acting at a distance according to mathematical laws.
>
> J. C. Maxwell, *A Dynamical Theory of the Electromagnetic Field*, 1864

Maxwell praised Weber's theory as 'ingenious and wonderfully comprehensive', but then went on to explain his alternative approach.

> The mechanical difficulties, however, which are involved in the assumption of particles acting at a distance with forces which depend on their velocities are such as to prevent me from considering this theory as an ultimate one, though it may have been, and yet may be useful in leading to the coordination of phenomena.
>
> I have therefore preferred to seek an explanation of the fact in another direction, by supposing them to be produced by actions which go on in the surrounding medium as well as in the excited bodies, and endeavouring to explain the action between distant bodies without assuming the existence of forces capable of acting directly at sensible distances.
>
> J. C. Maxwell, *A Dynamical Theory of the Electromagnetic Field*, 1864

While we can only guess what Maxwell might have made of the persistent problems in modern physics and the ideas presented in this book, recall that Maxwell wrote his poem *Recollections of Dreamland* that was quoted in Chapter 1, and he also originated the theory of *reciprocal space*, a dual way of describing normal space that was mentioned in Chapter 5, so who knows? Also remember that any measurement of a field depends on its interaction with matter particles, and all the information we can have about our world is mediated by electrons. Part of the job of fields is to convey energy and momentum, so when you pick up a pin by bringing a magnet close to it, the energy and momentum required to move the pin come from the flux of the electromagnetic field, but we also know that there are vast reserves of energy and momentum within particles, so how can we know that it does not actually come from these inner reserves?

Recall that we saw in Chapter 8 how the force of gravity is directed towards the instantaneous, NOW position of an attracting body, rather than the retarded position used in the calculations of electro-magnetism to allow for the delay of light speed propagation. Although gravitational waves have been observed and travel at the speed of light, the attractive force of gravity appears to act instantaneously. Physics does not provide any mechanism for this, just as it has no mechanism to account for the instantaneous propagation of entanglement information. But if all information in the inner world is available within the Compton time unit of a mass-carrying particle, there is no need for faster than light travel of information through space, the inner world offers the ultimate shortcut.

> Anyone who has contact with the supernatural is essentially sovereign, for in the form of the infinitely small he is a presence in society which transcends the social order.
> Simone Weil, *Gravity and Grace*

15
As Above So Below

The Golden Ratio

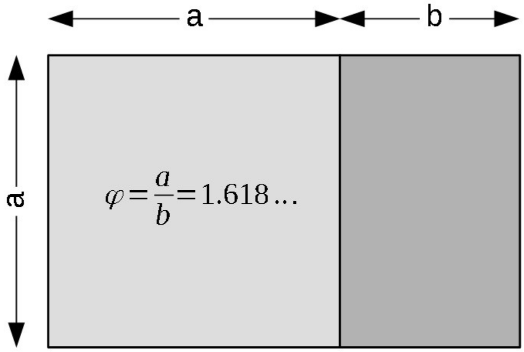


Fig. 15.1 The Golden Ratio φ

The number illustrated by the ratio of a to b in Fig. 15.1, is called the *golden ratio* φ, which is a special irrational number that obeys the equation:

$$\varphi + 1 = \varphi^2$$

φ has many magical properties that have fascinated mathematicians, artists, and architects since the time of Euclid. It also has many names, including the *golden mean*, *golden section*, *golden proportion* and *divine section*. There are two solutions to the equation for φ above, and after the decimal point, they have the same infinite progression of digits:

$$\frac{(1+\sqrt{5})}{2} = 1.618033\ldots \quad \text{and} \quad \frac{(1-\sqrt{5})}{2} = 0.618033\ldots$$

So here, once again, the unit 1 is acting as a sort of pivot between large and small. φ can also be written as an infinite progression like this:

$$\varphi=\frac{1}{\left(1+\frac{1}{\left(1+\frac{1}{\left(1+\frac{1}{(1+...)}\right)}\right)}\right)} \quad \text{or} \quad \varphi=\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+...}}}}$$

Yet another way to calculate φ is provided by the series of *Fibonacci numbers* shown below. Each term in the series is found by adding together the previous two numbers in the sequence. So, for example, 987 comes from adding 377 and 610:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, …

As the sequence gets longer, the ratio between successive numbers approaches ever closer to the golden ratio φ: For example, 5/3 = 1.66666..., 34/21 = 1.61904..., and 987/610 = 1.618032…

If the lengths a and b shown in Fig. 15.1 are wrapped around to form a circle, their golden ratio φ divides the circle with the golden angle of 137.5° shown in Fig. 15.2.

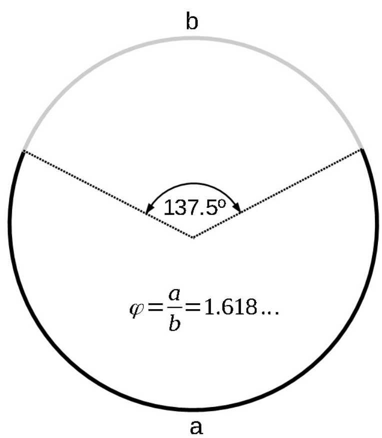


Fig. 15.2 The Golden Angle

The golden angle occurs throughout Nature, describing for example, the angle between one leaf of a plant and the next, in an arrangement that maximises their exposure to sunlight. An example is shown in Fig. 15.3, where the stem of each leaf points outwards at the golden angle relative to the stem of the previous leaf in a rotating sequence.

Fig. 15.3 The Golden Angle in Nature.

There is also a *golden spiral* which changes its radius by a factor of φ for every quarter turn. A very close approximation of a golden spiral can be drawn by fitting quarter circles into squares, where the side of each square is in proportion φ to the previous one; this is illustrated in Fig. 15.4.

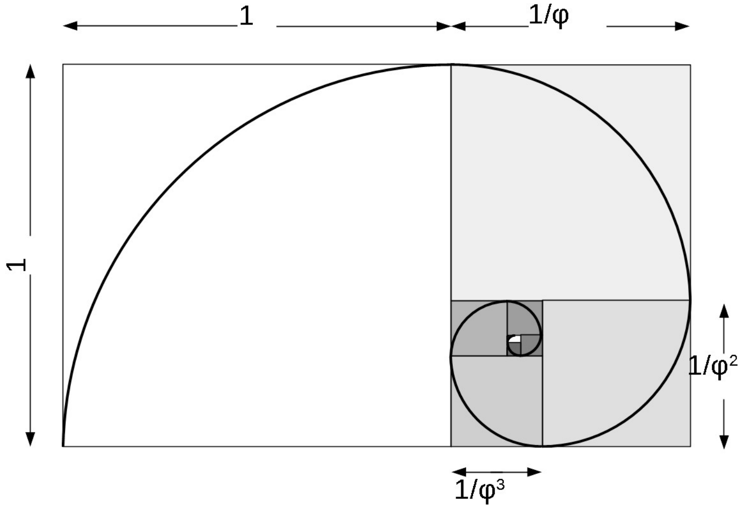


Fig. 15.4 The Golden Spiral

The Riemann Sphere

We saw in Chapter 4 how two-compartment, ‘complex’ numbers can be represented as points on 2D complex plane, and how multiplying by the imaginary unit i is equivalent to turning 90° around the unit circle, this was illustrated in Fig. 4.3. Complex numbers have reciprocals just as ordinary numbers do, so we can write the reciprocal z, of a complex number w:

$$z = 1/w, \qquad w = 1/z$$

Complex numbers are much richer mathematically than ordinary numbers, and offer a way of dodging the ‘divide by zero’ problem by extending the complex number plane in a perpendicular direction to include a point at infinity, and this gives the Riemann sphere, which Fig. 15.5 shows in dual forms.

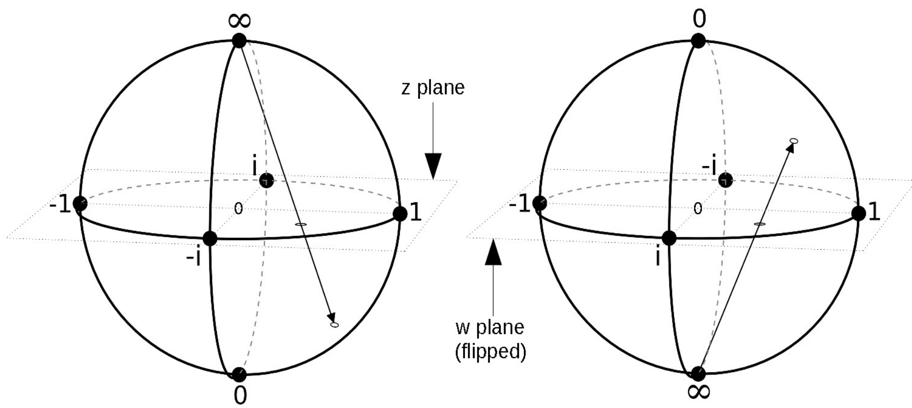


Fig. 15.5 The Riemann Sphere in dual forms.
Projecting information from 2D complex plane to 2D surface of the sphere.

The equatorial plane of a Riemann sphere is the usual complex plane containing a unit circle, but the plane can also be flipped over, and so can represent both a complex number w, *and* its reciprocal z = 1/w. Every point in the equatorial plane of the sphere can be assigned a coordinate in the w plane, and also a coordinate in the z plane, with the exception of the poles 0 and ∞. These two special points only have a coordinate in the w or z planes when they are zero, these 0s being marked at the centre of each unit circle. The ∞ in both the w and z coordinates is banished from the complex plane out onto the surface of the Riemann sphere.

In each representation of the Riemann sphere in Fig. 15.5, there is an arrow extending from the ∞ point, through a pixel in the unit circle on the equatorial plane, to pick out a pixel on the far side of the sphere. This is called *stereographic projection*, and by repeating the process for every pixel, an image of the whole equatorial plane can be projected up or down onto the opposite surface of the Riemann sphere. By projecting both up and down, all the directions in our familiar 3D space can be described by the complex numbers represented in 2D on the equatorial plane. The combination of the w and z pair of 2D representations on the equatorial plane can also describe 4D space-time, our 4D reality.

The Bloch Sphere

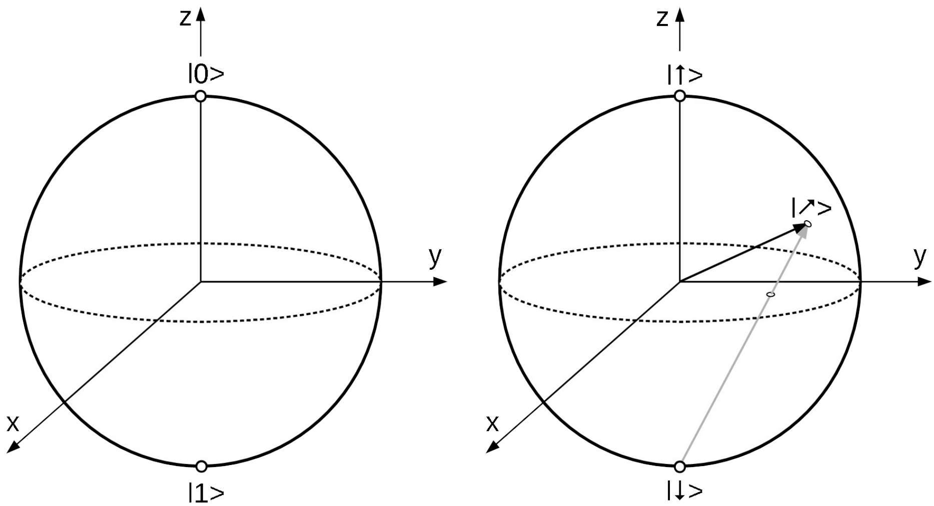


Fig. 15.6 The Bloch Sphere

Left: qubit base states 0 & 1. Right: qubit spin states ↑, ↓, and ↗.

In quantum mechanics, the Riemann sphere is known as the Bloch sphere, and is used to represent information. It is shown in Fig. 15.6. The ‘pure’ states that are available to a qubit before it is measured are represented as points on its surface, (there are also ‘mixed’ states that occur with qubits that have more complicated dependencies, and these are represented by points inside the sphere).

A qubit, as we saw in Chapter 9, can represent any quantum mechanical system with two possible base states, which could be 0 and 1, or spin up ↑ and spin down ↓, or some other pair of alternatives, and in the standard Dirac notation, these quantum states are written |0>, |1>, |↑> and |↓>. But a qubit can also be in any other pure state that is a superposition of its two base states, such as the state |↗> marked on the surface of the Bloch sphere in Fig. 15.6. This superposed state can be written as a sum of its two base states $|\nearrow\rangle = \alpha|\uparrow\rangle + \beta\,|\downarrow\rangle$, where the α and β are complex numbers describing the contribution of each base state to the superposition. The ratio of β to α, β/α, is also a complex number, and so it can also be represented on the complex equatorial plane of the Bloch sphere. Using the same stereographic projection as was shown in Fig. 15.5 for the Riemann sphere, the grey projection vector in Fig. 15.6 extends from the base state $|\downarrow\rangle$ at the bottom of the sphere, up through the point β/α on the equatorial plane, and out to the point representing the superposed state $|\nearrow\rangle$ on the sphere surface.

Electrons, protons and neutrons each have spin half, ½$\hbar$, so all of their pure spin states can be represented on a Bloch sphere. Notice this projection only works on the Riemann and Bloch spheres thanks to their use of dual descriptions, by both complex numbers *and* their reciprocals. While we have only considered the reciprocal transformation of a complex number here, more general transformations of complex numbers can do much more, such as mapping all that is represented *within* a unit circle, to a dual representation *outside* it. In *The Road to Reality*, Roger Penrose gives much more detail about the Riemann and Bloch spheres, and the magical properties of complex numbers.

Margolus-Levitin & Zitter

In their 1998 paper *The maximum speed of dynamical evolution*, Norman Margolus and Lev Levitin derived the fundamental limit that physics sets for the speed of computation, and this is known as the Margolus-Levitin theorem. They found the minimum time required for a quantum mechanical state (or wave function) to evolve to an orthogonal (completely distinct) state, and how the time taken depends on the energy E of the system. In his 2000 paper *Ultimate Limits to Computation*, Seth Lloyd also found that the maximum frequency of logical operations allowed by physics is $f_{max} = 2E/\pi\hbar$, which coincides with Margolus and Levitin's result, and showed that as well as NOT, other operations like AND, and FANOUT, can be performed at the same rate, while more complex operations that need to cycle through more than just two quantum states, are limited to half this speed, with $f_{max} = E/\pi\hbar$.

Taking the higher rate, and using $E = mc^2$ to convert energy to mass, we can write $f_{max} = 2mc^2/\pi\hbar$. Because π, $\hbar$ are all constants, this shows that the number of logical operations per second is directly proportional to mass, and gives a rate of 5.42 x 10^{50} operations per second per kilogram. Multiplying by an electron's mass of 9.11

x 10^{-31} kg, this gives $f_{max} = 4.94 \times 10^{20}$ operations per second for the maximum logical processing rate of an electron.

We saw in Chapter 13 that Schrödinger discovered zitterbewegung by playing with the Dirac equation, and found a zitter frequency of $2mc^2/\hbar$, or 1.55×10^{21} Hz. This is an angular frequency, and has to be reduced by a factor of 2π to get the frequency of complete revolutions (yes, I know, all these factors of 2π, and the distinction between the Planck h, and the reduced Planck $\hbar$, are very confusing for everybody, including me!). Doing this, we find the frequency of complete revolutions that an electron makes at its Compton scale to be 2.47×10^{20} Hz, exactly half the processing rate found by Margolus-Levitin and Lloyd. It seems that the maximum processing rate counts half revolutions at the electron's Compton scale.

From the above, it seems clear that the electron's zitter dance is the ultimate source of all the logical processing of the Cosmic Computer, at least at the higher levels of scale. But what happens at smaller scales, inside the nucleus of an atom, or even smaller, surely that is also logical processing? We know that the processing described by Margolus-Levitin is proportional to mass, and also that thousands of times more mass resides in the protons and neutrons of an atom's nucleus than in its orbiting electrons, so it is within the nucleus that most of the Cosmic Computer's processing must take place. This faster, inner processing, is included in the Margolus-Levitin accounting, despite its non-accessibility at higher scales. It is hard to make sense of this without postulating a fundamental information boundary in the Universe at the electron's Compton scale, the theatre of the zitter dance. Standard physicists might simply blame everything on the uncertainty principle, but perhaps their view needs a flip — the uncertainty principle may simply be a consequence of the electron's zitter dance, as David Hestenes suggested in his paper *The Zitterbewegung Interpretation of Quantum Mechanics*.

Sampling

Sampling is the process used to convert analogue information into digital information, and its most fundamental restriction comes from Shannon's sampling theorem that was mentioned in Chapter 10. Shannon showed that you can capture all the information in a signal, and convert it into digital bits, but only information carried by frequencies that are less than half your sampling frequency. This means that you need one sample for each up or down wiggle of the highest frequency in the signal. If higher frequencies are present, the information carried by them is not detected by the sampling, or, if it is, your digital data will be distorted by a process known as *aliasing*. When a number overflows its storage space in a digital register, it wraps from the highest number, back to the smallest number, this is the register overflow that was mentioned in the Introduction, and aliasing is closely related to this. The frequency limit before aliasing sets in is called the Nyquist frequency.

We know that all of the interactions in atoms are digital, and the digital unit is the Planck. We can look at the processes that go on inside an atom as a sampling of the outer world by the inner world, and vice versa. But frequencies only continue to get higher and higher as you go down in scale inside an atom, so the zitter frequency of an electron also appears to be a barrier, a Nyquist frequency, that limits what the outer world can know about the inner world — the uncertainty principle again? The ghostly shadows of signals that appear in the phenomenon of aliasing also seem reminiscent of the virtual particles that quantum field theories conjure up out of the vacuum at the scale of an atomic nucleus.

From the viewpoint of the inner world, looking outwards in the scale dimension, there would appear to be no problem in obtaining complete information about the outer world, because frequencies are lower in that direction and the Shannon limit would not apply. But is there anything in the inner world that could act as a memory to store this information? A memory does not necessarily have to possess rest-mass. In the early days of computing, devices called *delay lines* were used, which sent pulses of sound through tubes of mercury. The pulses were detected on arrival at the end of the tube, amplified, and then sent round again, and again…, and this provided a persistent memory. Material matter was needed to make these memories, but the information was carried by moving waves. Light waves can also be used for this purpose, and a photon can be seen as a memory unit travelling at light speed. The most widely used memory chips in computers are DRAM chips that 'forget' their contents so quickly that their memory has to be refreshed about fifteen times every second; hence their name Dynamic RAM. Process and memory are two sides of the same coin.

Filtering

Filtering is one of the main tasks of digital signal processing, DSP. A filter removes unwanted frequencies, or bands of frequencies, from a signal. In analogue AM radio broadcasting, a signal waveform is added to a carrier wave, and the signal is said to *modulate* the carrier wave. Although the two wave trains are added, modulation results in a superposition of *three* waveforms, the *carrier*, the *sum of signal and carrier*, and the *difference of signal and carrier*. This results in the broadcast waveband extending over two *sidebands*, one above the carrier frequency, and the other below, yet each carrying the same information. In analogue radio, it is common to filter out one of the sidebands, so as to take up less of the limited bandwidth of frequencies available, and this is called single sideband transmission. As the example in Chapter 10 shows, a Fourier transform gives a similar splitting of time domain information into dual positive and negative frequency representations in the frequency domain.

So here is another way of thinking about the Cosmic Computer: the information about all that is below the electron's Compton scale belongs to the higher frequency sideband of the Universe, and information about what is larger belongs to the lower sideband, in dual representations. The zitter, or Margolus-Levitin frequency of an electron, then appears as the carrier wave frequency, and the quantum measurement process, Penrose's **R** process, may be seen as a type of filtering.

Projecting Information

Very observant readers may have noticed that Fig. 12.3, which illustrates the idea of information written on a spherical surface, employs a form of golden spiral that wraps around the sphere. There is no known way of spacing things out perfectly evenly on the surface of a sphere, but the golden ratio can give the most even distribution possible for any number of bits, and the code for this is remarkably simple. I have seen two alternative ways of representing this surface information, one appears in *It from Bit*, John Wheeler's great rallying cry for a new physics based on information which appears in *Feynman and Computation*. Wheeler used a pattern like that made by latitude and longitude lines on a globe, but this has the problem that the 'squares' get more and more distorted as you go from the equator towards the north or south pole. Another way was used by Jacob Bekenstein in his *Scientific American* article of 2003, *Information in the Holographic Universe*, where he illustrated the information content of a black hole by tiling the surface with small triangles to represent the bits, but again this requires some distortion to make the triangles fit onto the sphere.

We saw above that a particle's inherent spin belongs to a separate inner world, and its direction can be represented on a Bloch sphere. This direction can also project itself out to influence real 3D world directions through the spin-orbit coupling between electrons and nuclear particles in atoms that seems to play an important role in sharing information between inner and outer domains. It is interesting to speculate that the golden ratio, angle, etc., that appear so frequently in our outer world, may be a natural consequence of the inner world arranging its outward projections in the most perfect way possible.

Gravity is non-local, and seems to be deeply connected with spin, so it may well also be the final arbiter of quantum measurement outcomes, as Penrose and others have suggested. Just as a Roman emperor indicated whether a gladiator should live or die by pointing his thumb up or down, the cogitations of the inner world may project outwards to influence real world outcomes. If Ed Fredkin is correct in his belief that the Universe is performing a calculation, perhaps the Cosmic Computer is just calculating π to great accuracy, or perhaps it is the impossible task of trying to describe the value of π with any finite number of digits, that ensures that the Cosmic Computer can never stop.

There is an important distinction between the NOT operations of ordinary computers, and the quantum NOT operations performed on qubits in quantum computers. While a normal bit can be flipped by two successive NOT operations and is returned to its original state, two successive NOT operations on a qubit cannot return it to exactly the state it started in. This is because a perfect NOT operation on a qubit is impossible, a consequence of the no-cloning theorem that forbids the making of an exact copy of a qubit. This also means that a zittering electron, while continually flipping its qubit in quantum NOT operations, must also evolve, staying within its unitary **U** process unless observed, but perhaps feeling subtle influences from gravity, or from its entanglement connections with other particles.

One complete cycle through the four components of Dirac's equation that describes the zitter, consists of four √NOT operations in succession, or two NOT operations. These two quantum NOTs are what guarantee the duality between outer and inner worlds, each always mirrors the other as closely as a quantum NOT will allow.

Dirac's Monopole

The quantum of negative electric charge carried by an electron is a *monopole*, an origin or sink for electric field lines that spread out like hedgehog spines in all directions, as illustrated on the left of Fig. 7.3. Field lines indicate the direction of flow of the flux of electromagnetic momentum and energy at any point. Electric field lines start on a charged particle and terminate on a particle of opposite charge. But magnetism does not seem to have any monopole sources or sinks where field lines can start or finish, it only has *dipoles* which the field lines pass straight through. A bar magnet is a dipole; if you cut it half way between its North and South poles, you just get two smaller dipole magnets, each with its own N and S poles. Magnetic field lines do not start or finish anywhere, they only circulate indefinitely, as illustrated on the right of Fig. 7.3.

The four Maxwell equations shown in Chapter 7 are completely symmetrical, which means that the electric **E** terms, and magnetic **H** terms, may be swapped without any consequence. But these 'vacuum equations' only apply to space which contains no sources of electricity or magnetism, and when extra terms are included to account for the presence of electric charges or currents, the equations lose their symmetry. However, if a monopole source of magnetism could exist, it would carry a magnetic 'charge', and the pleasing symmetry of Maxwell's equations could be restored.

As monopole magnetic sources had never been observed when Maxwell formulated his equations in 1864, he left them out of consideration. They continued to be more or less forgotten about by physicists until 1931, when Paul Dirac's highly influential paper *Quantum Singularities in the Magnetic Field* revealed their deep and essential role in quantum mechanics, and started a great search for monopoles that still continues, but without success. Theory suggests that a magnetic monopole would be absolutely stable, and could only be destroyed by encountering a monopole of opposite magnetic 'charge', producing an outpouring of their combined energy similar to that released by the annihilation of matter with anti-matter. Large numbers of monopoles should also have been created just after the Big Bang, so their total absence is another major mystery of physics.

Maxwell's equations work perfectly well without any magnetic sources, and unlike electric field lines which terminate on particles of opposite charge, magnetic field lines seem to be continuous. So how did Dirac conjure up his magnetic monopole?

> Paul Dirac ingeniously showed that the requirement for unbroken magnetic field lines doesn't rule out monopoles. Quantum mechanics allows them to exist, but only if their magnetic charge has exactly the right strength.
> Arttu Rajanie, *The Search for Magnetic Monopoles*, Physics Today 2016.

In the formula below, $\hbar$ is the reduced Planck, c is the speed of light, and 137 is the reciprocal of alpha, α, the fine structure constant that plays such a crucial role in the electron's zitter dance described in Chapter 13.

> The smallest charge is known to exist experimentally and to have a value *e* given approximately by

> The theory of this paper, while it looks at first as though it will give a theoretical value for *e*, is found when worked out to give a connection between the smallest electrical charge and the smallest magnetic pole. It shows, in fact, a symmetry between electricity and magnetism quite foreign to current views.
> Paul Dirac, *Quantum Singularities in the Magnetic Field*, 1931.

To obtain his results, Dirac analyses the behaviour of a wave function's *phase*. Every wave has its ups and downs, and a wave's phase just describes where it is in one of these up and down, or round and round cycles. He starts by writing the wave function Ψ for a particle in the form $\Psi = Ae^{i\gamma}$, a product of two components, where A describes the complex amplitude of the wave function, while γ describes the phase angle of $e^{i\gamma}$ as it cycles endlessly around the unit circle. Recall that the complex exponential $e^{i\gamma}$ passes through the four values 1, i, -1 and -i, in each trip around the unit circle, as we saw in Chapter 4, its value being multiplied by i with each quarter turn. In terms of the computing operations described in Chapter 9, the effect of these factors 1, i, -1, and -i, taking it in turns to multiply the wave function Ψ, is equivalent to enacting four √NOT logic operations in each cycle. A complete revolution of 360° is 2π when measured in radians, so every time the phase γ reaches 2π, it goes back to zero. There is no observable difference between a phase of γ, γ+2π, γ+4π, …, etc. The phase γ of a wave function is not measurable, and so it has no significance in the outer world. Because γ is indeterminate, any constant can be added to it without having any physical meaning, and only the difference between the values of the phase γ at different points is of any importance.

Besides being non-measurable, the phase generally depends on the path taken through the multi-dimensional mathematical space that quantum mechanics uses to represent a wave function, and this makes the phase non-integrable. If you measure the distance between two points by rolling a wheel and counting the complete rotations, not only will the result depend on the route taken, but so will the wheel's rotation angle, or phase, at the end point; the wheel's phase is also non-integrable.

Dirac first examines the conditions necessary for a wave function's phase to be able to vary without having any measurable effect in the outer world. He shows that the procedure of 'squaring the wave function' (Chapter 11), which multiplies Ψ by its conjugate Ψ* to get a real world probability ΨΨ*, ensures that their opposite changes of phase cancel each other out. While the wave function Ψ evolves forwards in time, its conjugate Ψ* evolves in reversed time, and multiplying them together as a product has the effect of freezing the phase time.

Dirac finds that phase uncertainty does not exist within a particle, but only arises when the particle is influenced by its surroundings.

> … all the general operations of quantum mechanics can be carried through exactly as though there were no uncertainty in the phase at all.
> ...As our dynamical system is merely a simple particle, it appears that the non-integrability of phase must be connected with the field of force in which the particle moves.
> ...We see that we must have the wave function Ψ always satisfying the same wave equation, whether there is a field or not, and the whole effect of the field when there is one is in making the phase non-integrable.
> Paul Dirac, *Quantum Singularities in the Magnetic Field*, 1931.

While the phase of the wave function is not definite at any point, the change of phase between any two points *does* have a measurable effect on outer world behaviour. Dirac expresses κ, the rate of change of phase, as a sum of four components: three, κ_x, κ_y and κ_z, for rates of change from motion in the 3 directions of space, and one, κ_0, for the rate of change in time. He is then able to derive an equation that describes how the total rate of phase change κ is distributed between its spatial components κ_x, κ_y and κ_z, and its time component κ_0, and this shows that a particle may *increase* its momentum in space, say in the x direction, by $h\kappa$, as long as its energy *decrease*s by $h\kappa_0$. A particle thus has the freedom to trade rates of phase change due to motion in space, with those due to energy, or 'motion' in time. These transactions are denominated in Planck sized units h, as we would expect, but the rates of change of phase κ are not necessarily integer values, so it would appear that subdivisions of the Planck unit h may play a role in the information trading at the sub-quantum phase level. Note that phase is a dimensionless number.

We saw in Chapter 8 that electromagnetism can be described not only by electric and magnetic fields, but also by means of a pair of potentials: a scalar potential which is oblivious to directions in space, and a vector potential that does take directional information into account. The fields are directly proportional to the rates of change of the potentials in space and time — the potentials belonging to a higher, *undifferentiated* view. Unlike the fields, however, the potentials couple directly to quantum wave functions, and Dirac takes advantage of this coupling. The potential formulation also brings in the new freedom of *gauge*: one potential is free to change as long as the other one makes a compensating change, and these changes of potential have no effect on the electric or magnetic fields.

> The connection between non-integrability of phase and the electromagnetic field given in this section is not new, being essentially just Weyl's Principle of Gauge Invariance in its modern form. … The present treatment is given only to emphasise that non-integrable phases are perfectly compatible with all the general principles of quantum mechanics and do not in any way affect their physical interpretation.
> … the non-integrable derivatives of the phase κ of the wave function receive a natural interpretation in terms of the potentials of the electromagnetic field...
> Paul Dirac, *Quantum Singularities in the Magnetic Field*, 1931.

When you measure an angle, you can add 360º (2π radians) to it as many times as you want, but you still come back to the same angle. Having established the connection between phase and gauge, Dirac continues:

> There is, however, one further fact which must now be taken into account, namely, that a phase is always undetermined to the extent of an arbitrary integral multiple of 2π. This requires a reconsideration of the connection between the κ's and the potentials and leads to a new phenomenon.
> Paul Dirac, *Quantum Singularities in the Magnetic Field*, 1931.

Dirac studies how a particle's wave function's phase changes after it has looped around a very small closed curve back to its starting point. Because the closed curve is so small, and the wave function is continuous, there is now no freedom for the phase to jump by arbitrary multiples of 2π, and he finds that the change in phase can be interpreted without ambiguity in terms of the electromagnetic flux through the small closed curve, and that this flux must also be small. But there is an exceptional case when the wave function vanishes, and the phase loses any meaning. Because the wave function is described by a two-compartment complex number, this can only happen when both its real and imaginary parts become zero, and because two conditions are required, all the points where the wave function vanishes as it passes through zero will lie on a line — a *nodal line*. (If you think of the waves in a vibrating string, nodes are the points along the string where it is still, while anti-nodes are the peaks and troughs of the motion.)

Each nodal line that passes through the small closed curve will contribute the same fixed amount to the electromagnetic flux passing through the curve, while increasing or decreasing the wave function's phase by 2π. For a closed curve in 3D space, only the magnetic flux needs to be considered, as the closure of the curve does not permit the divergence required by electric flux. Dirac now extends his analysis to a large closed curve, and then a closed surface. In general, a nodal line would cross a closed surface in 3D space twice, and the effects of the inward and outward crossings would cancel out, and leave the phase unaffected. But for any number n of nodal lines that do have an end point within the closed surface, each must cross the surface only once, and make an equal contribution to the flux through the surface. The common end point of all these n nodal lines would appear to be a singularity in the electromagnetic field — a magnetic monopole.

> ...at the end point there will be a magnetic pole of strength
>
> $$\mu = n\hbar c/2e$$
>
> Our theory thus allows isolated magnetic poles, but the strength of such poles must be quantised, the quantum μ_0 being connected with the electronic charge e by
>
> $$\mu_0 = 2/e\hbar c$$
>
> The theory also requires a quantisation of electric charge, since any particle moving in the field of a pole of strength μ_0 must have for its charge some integral multiple (positive or negative) of e, in order that wave functions describing the motion may exist.
> Paul Dirac, *Quantum Singularities in the Magnetic Field*, 1931.

Having established the link between the quantum of electric charge and the quantum of magnetic 'charge', Dirac continues:

> The wave functions discussed in the preceding section, having nodal lines ending on magnetic poles, are quite proper and amenable to analytic treatment by methods parallel to the usual ones of quantum mechanics. It will perhaps help the reader to realise this if a simple example is discussed more explicitly.
> Paul Dirac, *Quantum Singularities in the Magnetic Field*, 1931.

He uses the example of an electron moving in the magnetic field of a one quantum magnetic pole, and writes the equations using polar coordinates, expressed as radial distance from the pole, along with two angles that are equivalent to longitude and latitude on Earth. (Polar coordinates are a standard alternative to the x,y,z Cartesian coordinate system, but sometimes cause problems in calculations because they can make the origin appear to be a singularity.)

> It will not, however, be possible to obtain κ's satisfying these equations all round the magnetic pole. There must be some singular line radiating out from the pole along which these equations are not satisfied.
> Paul Dirac, *Quantum Singularities in the Magnetic Field*, 1931.

The equations now tell Dirac that there are no stable states for which the electron is bound to the magnetic pole, and that there must be two independent wave functions which represent opposite directions, one being continuous, the other discontinuous, with a jump of 2π in phase for any small closed curve encircling this direction. One way of thinking about this jumping is to imagine walking around the flagpole at the Earth's south pole. Each time you go round, you must cross the international date line, which is just one of all the lines of longitude that meet at the pole, and each time you step over the date line, your calendar date changes by plus or minus one day, equivalent to one complete 2π rotation of the Earth.

It is interesting that Dirac manages to conjure two wave functions out of one here, just as he found that four wave functions were needed to describe the behaviour of an electron according to his famous equation of Chapter 13. His nodal lines are now known as *Dirac strings*.

> In Dirac's model, [illustrated in Fig. 15.7], each magnetic north pole is connected to a magnetic south pole by a line of singularity called a Dirac string. That string is effectively an idealized solenoid with zero thickness, and it carries magnetic flux from the south pole to the north pole so that the field lines remain continuous. In classical physics, such a string would be easily observable because of the effect it would have on electrically charged particles. But in quantum physics, if the magnetic charge g of the poles has exactly the right value, electrically charged particles are unaffected by the string's presence.
> Arttu Rajanie, *The Search for Magnetic Monopoles*, Physics Today 2016.

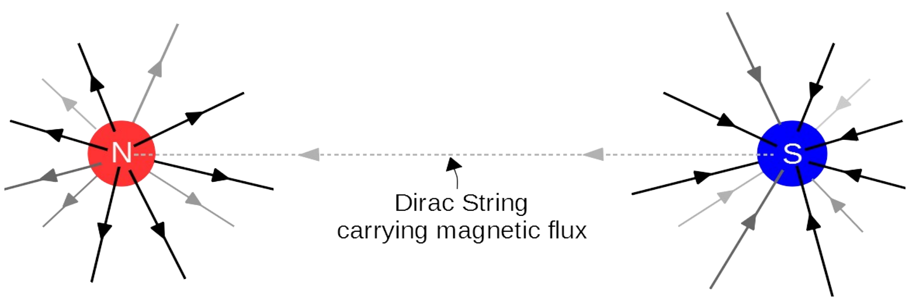


Fig. 15.7 Dirac string with Magnetic Monopoles as endpoints.
The Dirac string is unobservable, infinitely thin, and of unlimited length.

Dirac showed in this paper that physics, far from forbidding the existence of magnetic monopoles, actually seems to require at least one to exist, and this would be enough to determine the quantum unit of charge that is carried by every electron and proton in the Universe. He found that the value of the quantum of magnetic 'charge' g_0 *is* $2\pi\hbar/\mu_0 e$, which is 137/2 or 68½ times stronger than its electric counterpart. The greater strength of the magnetic 'charge' results in a force between two one-quantum magnetic poles of opposite sign that is $(68½)^2$ = 4692¼ times stronger than that between the opposite electric charges of an electron and a proton, and Dirac ends his paper with the suggestion: "This very large force may perhaps account for why poles of opposite sign have never yet been separated."

Dirac uses a wave function in three dimensions for his analysis, but recognises that his nodal lines would become 2D nodal surfaces in 4D space-time. He writes that "the passage to four dimensions makes no essential change in the theory", but this seems questionable if the atomic scale is a true discontinuity in 4D space-time. Perhaps each atom is just one pixel in an overall 2D nodal 'surface' of the Universe, and the zitter dance is like the action of a sewing machine needle going up and down through the surface of this Dirac sea.

Notice that it is the magnetic flux in the Dirac string of Fig. 15.7 that allows the magnetic monopoles to produce purely DIVergent hedgehog field lines like those produced by the electric source shown on the left of Fig. 7.3, and unlike the circulating CURL of the magnetic field lines shown on the right of Fig. 7.3. The Dirac string completes the circuit of magnetic flux, allowing it to flow out of the north pole and into the south pole. In concluding his paper Dirac writes:

> The object of the present paper is to show that quantum mechanics does not really preclude the existence of isolated magnetic poles. On the contrary, the present formulation of quantum mechanics, when developed naturally without the imposition of arbitrary restrictions, leads inevitably to wave equations whose only physical interpretation is the motion of an electron in the field of a single pole. This new development requires no change whatever in the formalism when expressed in terms of abstract symbols denoting states and observables, but is merely a generalisation of the possibilities of representation of these abstract symbols by wave functions and matrices. **Under these circumstances one would be surprised if Nature had made no use of it**. (My boldface)
>
> Paul Dirac, *Quantum Singularities in the Magnetic Field*, 1931.

Phase in Communications

We saw above that Dirac discovered his monopole by studying how the phase of a wave function depends on its separate spatial and time components. Communications engineers also use changes of phase to encode information in various ways on a carrier signal. The system used by GPS satellites to encode their data is called *binary phase-shift keying*, and this is illustrated in Fig. 15.8.

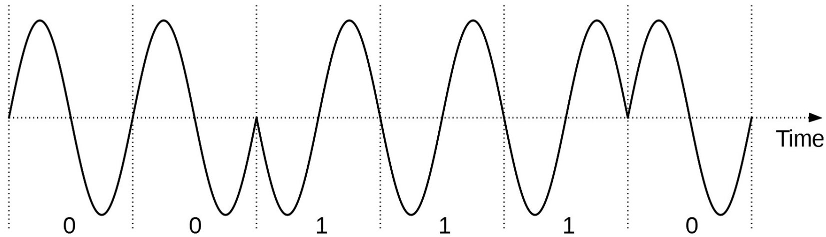


Fig. 15.8 Phase shift encoding.

You can see how the first two cycles of this signal are just pure sine waves and represent a 0, while in the next 3 cycles the phase is advanced by half a cycle, 180° (π radians), and this encodes a 1 value for those bits. The last of the 6 cycles is back in phase with the first two cycles, and so encodes a 0.

As phase shift encoding is widely used in our communications, it is reasonable to suppose that much of the information in the Cosmic Computer is held in phase relationships, and that shifts in the phase of an electron's quantum wave function carry information between the inner and outer domains.

A Magnetic Centre?

Dirac strings provide infinitely thin, secret passageways for magnetic flux that also point out directions in space. Dirac's analysis also shows that any number of quantum units of flux can originate or terminate at a monopole, so it is tempting to speculate that a magnetic monopole might somehow be concealed at the Planck scale, and that Dirac strings which originate there might run up through the scale dimension, through each particle, and out to the cosmic boundary. A proton occupies a scale 10^{20} times above the Planck, and has a surface area of 10^{40} Planck areas, so a Dirac string with a radius of the Planck length would have 10^{40} possible directions to choose from between the Planck scale and the surface of a proton. There are around 10^{80} protons, and the holographic cosmic surface has around 10^{120} Planck areas, so each proton would also have 10^{40} external directions available. According to the holographic principle, each Planck area on the cosmic surface represents about one bit of information, so perhaps these are also terminations of Dirac strings, but represented on a surface which offers a 'blown up' view of the Planck scale. When looked at like this, every material object in the Universe would be like a puppet suspended on cosmic strings.

> Oh Arjuna! There is nothing higher than Me; all is strung upon Me as rows of pearls upon a thread.
> Bhagavad Gita VII

The ideas above are pure speculation, but may suggest ideas for future research. Science is always an unfinished story, and our understanding of the Cosmic Computer and its relation to the Perennial Philosophy is just beginning. We certainly need a better understanding of the electron's zitter dance, and the role that a Dirac monopole may play in it…

> Around 1950, I had the rare opportunity of meeting Albert Einstein … The professor addressed my colleague: '*Vot* are you studying?' 'I'm doing a thesis on quantum theory'. '*Ach!*' said Einstein, 'a *vaste* of time!'
> He turned to me 'And *vot* are you doing?' I was more confident: I'm studying experimentally the properties of pions'. 'Pions, Pions, *Ach, vee* don't understand *de* electron! V*y* bother *mit* pions?'...
> Leon Lederman: *Life in Physics and the Crucial Sense of Wonder*. CERN Courier, 10 September 2009.

The Principles of Hermes

The principles of Hermes were set out by Hermes Trismegistus in around 100 AD, and are part of the Perennial Philosophy. We will go through them one at a time, to see how well they agree with the perspective that has been set out in this book.

1) The All is mind; the Universe is mental.
2) As above, so below, as below, so above. As within, so without, as without, so within.
3) Nothing rests; Everything moves; Everything vibrates.
4) Everything is dual, everything has poles and everything has its pair of opposites.
5) Everything flows, out and in; Everything has its tides; All things rise and fall.
6) Every cause has its effect; Every effect has its cause.

7) Gender is in everything; Everything has its masculine and feminine principles.

1) The All is mind; the Universe is mental. We have seen that Yoga philosophy ranks *Chitta* (mind-substance) at the highest level in the order of manifestations of the Absolute, above *Prana* (energy), and *Akasa* (matter). In physics, the Margolus-Levitin theorem allows all physical processes to be understood as logical information processing. We have noted the basic simplicity of the underlying components of computers, and how the properties of components at small scales seem to defy the normal rules of engineering, with everything getting better as you go smaller, and this led to the silicon chips that have changed our world. Nanotechnology is improving rapidly, and a nanometre, 10^{-9} m, is only ten times bigger than the size of an atom, so we are rapidly approaching the atomic scale.

We cannot make the components required to build quantum computers, we have to rely on natural quantum components like the atoms and photons provided by Nature, and harness their unique quantum powers. Whereas conventional computers run in serial fashion and perform just one operation at each step, a quantum computer can explore many operations in parallel at each step.

We no longer need to instruct computers in tedious detail about how to perform many tasks, AI machines can teach themselves from experience, and even discover new ideas. AI can be seen as a threat to humanity, taking over and destroying or enslaving humans, but AI can also be seen as a boon, and it is encouraging to hear players of Chess and Go enthuse about wonderful new strategies being discovered by AI that have eluded human players for thousands of years. Unlike traditional programmers who have to fuss over every symbol in their code to get a program to work, AI machines are trained by example, learning how to best organise their vast number of internal connections so that input data is routed to the correct output. The creators of AI machines cannot say in detail how their logic works, there is far too much going on inside the AI, so it appears random, just like a physicists' interpretation of outputs from the quantum world.

2) *As above, so below, as below, so above. As within, so without, as without, so within*. The second principle of Hermes is expressed in this book as the duality of information in the dimension of scale, which offers a natural home for the dualities of above↔below, and without↔within. The Planck quantum of action h is a fixed quantity, but its component dimensions of ML^2/T give it an internal flexibility that prevents it being pinned down to a particular length or time scale. Angular momentum has the same dimensional makeup as action, and this also allows a certain

flexibility — an ice skater can change her rate of spin by moving her arms in or out, but her angular momentum must remain constant. We know that the Planck unit acts as a sort of central roundabout for information in quantum theory, and that the electron, which embodies a quantum of charge, a quantum of magnetic flux, and a quantum of angular momentum, is the principle actor in the play of reality.

3) *Nothing rests; Everything moves; Everything vibrates*. Physics has two types of particle, bosons, and fermions. Photons are bosons which are unable to stand still, they must always travel at the speed of light. The mass-carrying particles that make up matter are called fermions, and can stand still, but they are best described by the Dirac equation, which shows that *internally* all is moving at light speed, and it is only the average of these back and forth, and round and round, inner motions that makes them appear to be at rest when viewed from outside. The internal zitter dance of an electron can also be seen as motion forwards and backwards in time, all contained within a Now defined by the Planck.

4) *Everything is dual, everything has poles and everything has its pair of opposites*. Duality in the scale dimension was already considered under principle 2, but we have seen many more dualities, and we noted that Shahn Majid sees duality as a meta-principle of physics. As we saw, the dual representations provided by Fourier transforms are ubiquitous in physics and engineering, and play a central role in quantum mechanics. The humble bit, 0 or 1, is a fundamental duality that was recognized by George Boole in 1854 as being the essential basis for logical thought. Its modern twin, the qubit, includes a further duality that allows it to not only behave like a bit and be 0 or 1, but also be part 0 and part 1 simultaneously.

5) *Everything flows, out and in; Everything has its tides; All things rise and fall*. Quantum mechanics offers a wave picture of matter, which is dual to the particle picture, and describes everything as rising and falling flows of energy. Bohm described the 'breathing' of the Universe as the unfolding and enfolding of information between his implicate and explicate orders. The zitter dance of an electron involves energy oscillating between one form and another like waves that bob up and down on the Dirac sea.

6) *Every cause has its effect; Every effect has its cause*. We have seen that the law of cause and effect is unquestioned in eastern thought, and also in the Perennial Philosophy, it is only 20th century physics that abandoned it and embraced the god of randomness — physics needs a 21st century update, perhaps it is lame because it has abandoned all religious principles. Feynman's *sum over histories* version of quantum mechanics makes it plain that, in the quantum world, Nature explores all

possibilities simultaneously, it is only the restrictions of the outer world that limit what we experience in reality. But physicists are taught that outer world outcomes of quantum processes are devoid of any meaning, and so blind themselves to much of the exquisite architecture and operation of the Cosmic Computer.

7) *Gender is in everything; Everything has its masculine and feminine principles*. We might see the *globalised* totality of the Universe as the feminine principle, the mother, the womb of creation that supports the life of all beings. We might also see the *localised* singularities of mass-carrying particles and the Planck point as the male principle, the father, the source that seeds all creation in our Universe.

The AUM mandala makes it clear that the electric charge in every atom is separated in the scale dimension: all the positive charge is locked up in the nucleus, while all the negative charge is found in the orbiting electrons — the Universe is electrically polarised in the scale dimension. So we might see the positively charged inner world as feminine, and the negatively charged outer world as masculine. The Dirac equation also reveals that electrons are secretly bisexual, continually flipping their charge, but at a rate so fast that their secret is withheld from the outer world. The zitter dance that electrons perform may also be seen as the cycling of yin and yang, or the cosmic tango of Shiva with Shakti, with the partners taking it in turn to lead the dance.

> According to Shaivism, one of the major branches of yogic philosophy, there is a divine masculine energy that takes the form of the Hindu god, Shiva, and a divine feminine energy that takes the form of the goddess, Shakti.
>
> Both Shiva and Shakti are alive in both men and women. All of us have divine masculine (Shiva) aspects and divine feminine (Shakti) aspects to our being.
>
> It's said that our feminine side resides on our left side, while the masculine resides on our right side. We hold these energies within us and, when united, there's a complete balance, joy and presence within our very being.
> Yogapedia https://www.yogapedia.com/shiva-and-shakti/2/6052

Appendix 1

An example of Boole's Method from *The Laws of Thought*

Boole's words are indented and written in *italics*.

> *Let us take as an example the definition of "clean beasts," laid down in the Jewish law, viz., "Clean beasts are those which both divide the hoof and chew the cud," and let us assume*
>
> *x = clean beasts;*
>
> *y = beasts dividing the hoof;*
>
> *z = beasts chewing the cud.*
>
> *Then the given proposition will be represented by the equation*
>
> *x = yz*
>
> *which we shall reduce to the form*
>
> *x - yz = 0,*
>
> *and seek that form of interpretation to which the present method leads.*

Now Boole 'develops' the symbols of the expression x - yz that he wants to interpret in two stages. First he writes every possible true and false combination of the symbols. As there are three symbols, each with the two possible values (e.g. y means 'beasts dividing the hoof' and (1-y) means 'beasts not dividing the hoof'), he produces eight terms in total:

xyz, xy(1-z), x(1-y)z, x(1-y)(1-z), (1-x)yz,

(1-x)y(1-z), (1-x)(1-y)z, (1-x)(1-y)(1-z)

For each of these eight terms, he now, in the second stage, needs a factor to multiply it by. He gets these factors (which I write in **grey**) in the following way. For the first term, xyz, the x, y and z are all in their 'true' form and so he substitutes 1 for each of them into x – yz (the expression he found above) to get 1 - 1, which equals 0, so this is his factor for multiplying the first term to give **0**xyz . For the second term, xy(1-z), he substitutes into x - yz, 1 for x, 1 for y, and 0 for z, as the z component is in the form (1-z), to get 1 - 0 = 1, so the second term becomes **1**xy(1-z). For the third term,

$x(1-y)z$, he substitutes into $x - yz$, 1 for x, 0 for y, as it is in the form $(1-y)$, and 1 for z, to get $1 - 0 = 1$, so the third term becomes $\mathbf{1}x(1-y)z$. The other factors are found in the same way, and he ends up with:

$$\mathbf{0}xyz, \quad \mathbf{1}xy(1-z), \quad \mathbf{1}x(1-y)z, \quad \mathbf{1}x(1-y)(1-z), \quad \mathbf{1}(1-x)yz,$$

$$\mathbf{0}(1-x)y(1-z), \quad \mathbf{0}(1-x)(1-y)z, \quad \mathbf{0}(1-x)(1-y)(1-z)$$

All the terms pre-multiplied by **0** now vanish and he has the four terms pre-multiplied by **1** remaining. Each of which he equates separately to 0:

$$xy(1-z) = 0, \qquad xz(1-y) = 0, \qquad x(1-y)(1-z) = 0, \qquad (1-x)yz = 0$$

He is now ready to interpret these equations:

> *These equations express a denial of the existence of certain classes of objects, viz.:*
>
> *1st. Of beasts which are clean, and divide the hoof, but do not chew the cud.*
>
> *2nd. Of beasts which are clean, and chew the cud, but do not divide the hoof.*
>
> *3rd. Of beasts which are clean, and neither divide the hoof nor chew the cud.*
>
> *4th. Of beasts which divide the hoof, and chew the cud, and are not clean.*
>
> *Now all these several denials are really involved in the original proposition. And conversely, if these denials be granted, the original proposition will follow as a necessary consequence. They are, in fact, the separate elements of that proposition. Every primary proposition can thus be resolved into a series of denials of the existence of certain defined classes of things, and may, from that system of denials, be itself reconstructed. It might here be asked, how is it possible to make an assertive proposition out of a series of denials or negations? From what source is the positive element derived? I answer, that the mind assumes the existence of a universe not a priori as a fact independent of experience, but either a posteriori as a deduction from experience, or hypothetically as a foundation of the possibility of assertive reasoning.*

Boole's original method is rather strange and mysterious to those of us familiar with modern Boolean logic. The main point I want to emphasise is that he always uses the background, the NOT something, as well as the something; his method takes into account ALL the possibilities. Quantum information processing also explores ALL possibilities.

In the problem analysed by Boole's method above, the **grey** factors only took on the values 0 and 1. But more generally, in other problems of logic, four other possible factors of this type arise that are ratios of 0 and 1:

$$\tfrac{0}{0} \qquad \tfrac{0}{1} \qquad \tfrac{1}{0} \qquad \tfrac{1}{1}$$

As we might expect, he equates the $\tfrac{0}{1}$ to 0, and the $\tfrac{1}{1}$ to 1.

Strangely, he also equates $\tfrac{1}{0}$ to 0:

> *. . . $\tfrac{1}{0}$ is the algebraic symbol of infinity. Now the nearer any number approaches to infinity (allowing such an expression), the more does it depart from the condition of satisfying the fundamental law above referred to. . . any constituent whose coefficient is not subject to the same fundamental law as the symbols themselves must be separately equated to 0.*

So because $\tfrac{1}{0}$ breaks his fundamental Law of Thought that only permits numbers to be 0 or 1, he sets $\tfrac{1}{0}$ equal to 0!

To the $\tfrac{0}{0}$ he gives a special interpretation:

> *The coefficient $\tfrac{0}{0}$ indicates that a perfectly indefinite portion of the class, i.e. some, none, or all of its members are to be taken.*

I find it fascinating that Boole's system of logic encompasses not just the 1-x, the NOT something, but also the unknowable — and this unknowable $\tfrac{0}{0}$ is 'real' enough to have an effect, and be interpretable in his system. Perhaps Boole should be credited with discovering the qubit as well as the bit?

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I have shelves full of books on physics, both popular and technical-mathematical, and while all have helped me and given particular insights, here are a few that have been particularly indispensible:

The Feynman Lectures on Physics can be read for pure pleasure, quite apart from Feynman's stunningly simple presentation of the deepest subjects. They are also available online for free.

The Feynman Lectures on Computation were given by Feynman in 1962-3, and are a wonderful introduction to all sorts of computers and the physics of information.

Feynman and Computation has a few contributions from Feynman, but mainly consists of chapters by other leading researchers in the field. It is full of interesting ideas and insights.

The World According to Wavelets by Barbara Hubbard offers the best introduction to Fourier Analysis that I have found. Nearly all texts on this subject are just of impenetrable mathematics. This has a plain english section that anyone can understand and appendices containing all the rigorous mathematics. It opened the door to this vital subject for me.

The Variational Principles of Mechanics by Cornelius Lanczos is the classic text on analytical mechanics, the alternative to Newtonian mechanics derived from the principle of Least Action. Lanczos has an almost religious passion for the subject.

The Road to Reality by Roger Penrose is a complete guide to physics, and a key source for much of the physics in the book your are reading, but be prepared for a lot of mathematics.

The Laws of Thought by George Boole (1854) was described by Bertrand Russell as the birth of pure mathematics. Boole invented the first way of using algebra to perform logic, but his method was not what we now call Boolean algebra. His law of thought is $x = x^2$, which is only satisfied by the two numbers 0 and 1 that he called Nothing and Universe. Inspiring, but much of the book consists of tedious examples of his method.

On Space and Time by Connes, Heller, Majid, Penrose, Polkinghorne and Taylor, is full of deep insights into physics. As Heller and Polkinghorne are both Christian theologians as well as being physicists, their contributions clarify the connection between physics and spirituality.

Q.E.D. is Feynman's amazing explanation of quantum electrodynamics without mathematics, he just draws little arrows.

Programming the Universe, by Seth Lloyd, is an MIT professor's description of the universe as a cosmic computer. Lloyd's work is the basis for much of the book you are reading.

Black Holes and Time Warps, by Kip Thorne is an account of the history of gravitational research in the second half of the 20th century by one of its leading researchers. It is a pleasure to read, offering deep insights into the physics, and the struggles of physicists to understand Einstein's legacy.

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